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Wentworth. Answers to Algebra .

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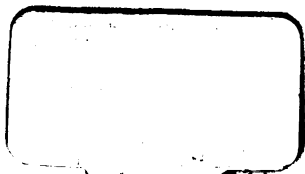
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ANSWERS

WENTWORTH'S

ELEMENTS OF ALGEBRA

AND

COMPLETE ALGEBRA

BOSTON, U.S.A.

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ANSWERS.

Exercise 1. Page 12.

- | | | | | |
|--|----------|----------|-----------------------|-----------------------|
| 1. 22. | 6. 94. | 11. 6. | 16. 25. | 21. $39\frac{3}{4}$. |
| 2. 26. | 7. 81. | 12. 1. | 17. $64\frac{3}{4}$. | 22. $20\frac{1}{2}$. |
| 3. 564. | 8. 16. | 13. 120. | 18. 31. | 23. $20\frac{1}{2}$. |
| 4. 6. | 9. 2. | 14. 25. | 19. -15. | 24. 0. |
| 5. 39. | 10. 127. | 15. 43. | 20. 24. | 25. $10\frac{1}{2}$. |
| 26. $a+b$ 27. $2x$ 28. $a-5$ 29. $x+1$. | | | | |
| 30. $x-2, x-1, x, x+1, x+2$. | | | | 32. y . |
| 31. ax . | | | | 33. $a-b$. |
| 34. $ab+c$ 35. $\frac{x+y}{c} = abm - 6c + \frac{a}{x+y}$. | | | | |
| 36. $\frac{6n^2}{m-a} + 5b(c+d-a)$ 37. $4a^4 - 5a^2b^2 + 3b^4$. | | | | |

Exercise 2. Page 14.

- | | |
|--|--|
| 3. 30, 5. | 5. 60 ft. broken off; 15 ft. standing. |
| 4. 25 cents. | 6. 27, 36. |
| 7. Horse, \$168; cow, \$42; sheep, \$6. | |
| 8. Apples, 13 cents; pears, 26 cents; oranges, 52 cents. | |
| 9. Harness, \$50; wagon, \$100; horse, \$200. | |
| 10. Thomas, \$1.20; Richard, \$1.20; Henry, 60 cents. | |
| 11. A, \$100; B, \$400; C, \$500. | |

Exercise 3. Page 17.

- | | | | |
|--|---------|---------|--------------|
| 1. 5. | 3. -11. | 5. 51. | 7. \$2484. |
| 2. -40. | 4. -3. | 6. 495. | 8. B.C. 241. |
| 9. A.D. 14. 10. 150 steps in all; 76 steps forward. | | | |

Exercise 4. Page 18.

- | | | | |
|---------------|-------------------|-----------------------|------------------|
| 1. 0. | 4. $3x^2$. | 7. $25my$. | 10. $14a^2x^2$. |
| 2. $6mx$. | 5. $7my^2$. | 8. $-22ab^2$. | 11. $6b^2m^3$. |
| 3. $-20mng$. | 6. $2ab$. | 9. $-55xy$. | 12. $9a-10b$. |
| | 13. $-a^2c+xyz$. | 14. $3x^2y-4ab-2mn$. | |

Exercise 5. Page 19.

- | | | |
|----------------------------------|--|------------------------|
| 1. $9a+9b+9c$. | 3. $2a+2b$. | 5. $-2a+2b+2d$. |
| 2. $a-b$. | 4. $a+b+c$. | 6. $2x^3-9x^2-8x+10$. |
| 7. $5x^4+4x^3+3x^2+2x-9$. | 9. $6ab+7ax^2-9a^2x+ax^3$. | |
| 8. $4a^3-4ab^2+2a^2b-7b^2+b^3$. | 10. $-c^4+c+8$. | |
| | 11. $-2xy-2y^2+11yz+5z^2-6y-6z$. | |
| | 12. $6m^5-m^4n+2m^3n^2+3m^2n^3-4mn^4+5n^5$. | |

Exercise 6. Page 21.

- | | | | | |
|-------|--|---------|---------|---------------|
| 1. 9. | 2. -25. | 3. -89. | 4. 200. | 5. 506 years. |
| | 6. 35° in latitude ; 75° in longitude. | | | |

Exercise 7. Page 22.

- | | | |
|-----------------|--------------------|------------------|
| 1. $9x$. | 6. $41ax^3$. | 11. $6x^2$. |
| 2. $-8ab$. | 7. $8a^2x$. | 12. $13x^2y$. |
| 3. $-7ab^2$. | 8. xy . | 13. $-6ax^3$. |
| 4. $22m^2x^2$. | 9. $8ax+3ay$. | 14. $-3ab+6mx$. |
| 5. $-4ay$. | 10. $2ab^2y-aby$. | 15. $3a-2b+4c$. |

Exercise 8. Page 23.

- | | | |
|------------------------|---|------------------|
| 1. $4a+2c$. | 2. $a+6b+4c$. | 3. $2x^2-2x-4$. |
| 4. $3x^4-x^3-14x+18$. | 6. $-5xy-5xz+yz+2y^2$. | |
| 5. $x^3-ax+2a^2$. | 7. $2a^3-6a^2b+6ab^2-2b^3$. | |
| | 8. $-4xy+2xz-y^2+5yz-z^2$. | |
| | 9. $ax^2+7abx-bx^2+b^2x+12b^3+x^3$. | |
| | 10. $-2x^3+3xy^2-y^3-14x^2+2xy-10y^2+6$. | |
| 11. $-a^4-8ab^3+b^4$. | 12. $-2x^5+4x^4y-8xy^4-8y^5$. | |

13. $a^2b^2 - 3a^2bc - 3ab^2c - abc^2 - a^2c^2 - b^2c^2$.
14. $2a + 4b - 9c + d$. 15. $b - a; a^3 + a^2b + 6ab^2 + b^3$.
16. $x^2 - 4xy + 2z^2$. 18. $-10a^3 - 4ab + c^2$.
17. $19ac + 17cd - 17$. 19. $xy - 2x - 6y + 1$.
20. $-2a^2bc - 2ab^2c + 2abc^2 - 2abc$. 21. $5x^2 - 5x + 5$.
22. $x^2 + xy + y^2$. 24. $2bx + 2by$.
23. $ax^2 - by^2 - cx^2 + dy^2$. 25. $7x - 7y + 2y^2$.
26. $a^2b^2 + 12abc - 9ax^2 - 4ab^2 + 6acx - 3a^2x$.
27. $-a^2 - 6b^2 + c^2$. 29. $x^2 - 3y^2 - 3z^2$.
28. $2a^3$. 30. $2a^3 - 2a^2c + 2ac^2$.

Exercise 9. Page 26.

1. $2b$. 7. $a + a^2$. 13. $-b + 10c$.
2. $a + b - 2c$. 8. $2a + 3c$. 14. $-5a$.
3. $3x - 3y - z$. 9. $3a - 3b$. 15. $4a - 16b - 2c$.
4. $a - b + c$. 10. $2a - b$. 16. $3a - 2c$.
5. $-2x$. 11. $5a$. 17. $16 - 12x$.
6. $-9b + 14c$. 12. $4x - y - z$. 18. $4c$.
19. $3a$. 20. a .

Exercise 10. Page 27.

1. $(2a - 3b) - (4c - d) + (3e - 2f);$
 $(2a - 3b - 4c) + (d + 3e - 2f).$
2. $(a - 2x) + (4y - 3z) - (2b - c);$
 $(a - 2x + 4y) - (3z + 2b - c).$
3. $(a^5 + 3a^4) - (2a^3 + 4a^2) + (a - 1);$
 $(a^5 + 3a^4 - 2a^3) - (4a^2 - a + 1).$
4. $-(3a + 2b) + (2c - 5d) - (e + 2f);$
 $-(3a + 2b - 2c) - (5d + e + 2f).$
5. $(ax - by) - (cz + bx) + (cy + az);$
 $(ax - by - cz) - (bx - cy - az).$
6. $(2x^5 - 3x^4y) + (4x^3y^2 - 5x^2y^3) + (xy^4 - 2y^5);$
 $(2x^5 - 3x^4y + 4x^3y^2) - (5x^2y^3 - xy^4 + 2y^5).$

7. $\left\{ \begin{array}{l} 1. (2a - 3b - 4c) + (d + [3e - 2f]). \\ 2. (a - 2x + 4y) - (3z + [2b - c]). \\ 3. (a^5 + 3a^4 - 2a^3) - (4a^2 - [a - 1]). \\ 4. -(3a + 2b - 2c) - (5d + [e + 2f]). \\ 5. (ax - by - cz) - (bx - [cy + az]). \\ 6. (2x^5 - 3x^4y + 4x^3y^2) - (5x^2y^3 - [xy^4 - 2y^5]). \end{array} \right.$
8. $(2a - 4b - 2c)x - (6a + 3c)y + (4b)z.$
 9. $(a - b)x + (2a + 3)y + (4a - 3b - 2)z.$
 10. $(a - 4b - 2c)x - (2b + 3c + a)y + (a + 4b + 5c)z.$
 11. $(12a - 15c)x + (12a + 4b + 6c)y - (12b - 3c)z.$
 12. $(2a - 2b)x - (3b + c)y.$

Exercise 11. Page 29.

- | | | |
|-------------|---------------|--------------------|
| 1. -136. | 5. -258. | 9. -728. |
| 2. -320. | 6. 45,700. | 10. 2052. |
| 3. 18.0621. | 7. 61,225. | 11. 183.92. |
| 4. 90. | 8. 82.477956. | 12. -1718.382762. |
| | | 13. -8.0001703416. |

Exercise 12. Page 30.

- | | | |
|-----------------|------------------------|-------------------------|
| 9. $-12a^3.$ | 15. $-10a^{m+n}.$ | 21. $-42m^4qx^3.$ |
| 10. $45m^2n.$ | 16. $21a^5x^6.$ | 22. $-144p^6q^6.$ |
| 11. $-12abxy.$ | 17. $224abc.$ | 23. $24a^6m^9x^8.$ |
| 12. $-8cdmn.$ | 18. $-96a^2b^2c^3.$ | 24. $162x^8y^4z^6.$ |
| 13. $-14a^2bc.$ | 19. $-18,954a^2bmp^2.$ | 25. $-24a^2b^2m^2x^2.$ |
| 14. $15m^3x^3.$ | 20. $-60a^3b^5y^7.$ | 26. $-168a^4b^6m^3n^6.$ |

Exercise 13. Page 32.

- | | |
|---|---|
| 5. $12a^3b - 9ab^2.$ | 7. $6x^4y - 8x^2y^3 + 10x^2yz^2.$ |
| 6. $24a^4 - 27a^3b.$ | 8. $a^4x^3y - 5a^3x^4y + a^2x^5y + 2ax^6y.$ |
| 9. $27a^6b^4 - 9a^4b^6 + 12a^3b^7 + 3ab^9.$ | |
| 10. $-15x^5y + 10x^4y^2 + 35x^3y^3 - 5x^2y^4.$ | |
| 11. $12x^3y^3 - 15x^4y^2 - 24x^5y.$ | 12. $3a^4 - 2a^3b - a^2b^2.$ |
| 13. $3x^3yz^2 + 6x^4yz^3 - 15x^5y^2z^3 + 18x^6y^3z - 9x^6y^3z^2.$ | |

Exercise 14. Page 34.

1. $x^4 + x^2 - 20$. 4. $x^3 - y^3$. 7. $x^4 - 2x + 1$.
2. $y^2 + 7y - 78$. 5. $2x^2 + 3xy - 2y^2$. 8. $x^3 - 9a^2x$.
3. $a^6 - x^6$. 6. $6x^4 - 96$.
9. $-10b^3 - ab^2 + 26a^2b - 7a^3$.
10. $2a^2 + 5ab + 2b^2$. 13. $6a^3b - 23a^2b^2 + 20ab^3$.
11. $a^3 - b^3$. 14. $-4a^5 - 4a^2b^3 + 16a^3b^2 - 8ab^4$.
12. $a^3 + b^3$. 15. $a^4 + a^2b^2 + b^4$.
16. $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$.
17. $x^2 - 4y^2 + 12yz - 9z^2$.
18. $6x^4 + x^3y - 16xy^3 + 2x^2yz + 3xy^2z + 4y^3z$.
19. $x^4 + x^3y + x^2y^2 + x^3z + x^2yz + xy^2z + x^2z^2 + xyz^2 + y^2z^2$.
20. $a^3 - 3abc + b^3 + c^3$. 22. $x^5 + 151x - 264$.
21. $x^3 + 3xy + y^3 - 1$. 23. $x^5 - 41x - 120$.
24. $x^6 + 10x - 33$.
25. $x^7 - 7x^6 + 21x^5 - 17x^4 - 25x^3 + 4x^2 + 2x$.
26. $4x^6 - 5x^5 + 8x^4 - 10x^3 - 8x^2 - 5x - 4$.
27. $25a^7b - 9a^5b^3 + 22a^4b^4 - 4a^3b^5 + ab^7$.
28. $4a^{13}y^3 + 8a^{11}y^4 - 64a^5y^7 - 128a^3y^8$.
29. $6m^7n - 18m^5n^3 + 18m^3n^5 - 6mn^7$.
30. $24a^3b - 26a^5b^5 + 4a^4b^6 - 6a^3b^7 - 5a^2b^8 + 4ab^9 - 3b^{10}$.
31. $x^4 - 10x^2 + 9$. 32. $x^3 + x^4 + 1$. 33. $a^8 + a^4b^4 + b^8$.
34. $32a^7b - 8a^5b^2 + 8a^5b^3 - 6a^4b^4 - 2a^3b^5 - a^2b^6 + ab^7$.
35. $x^5 + ax^4 - 27a^2x^3 - 13a^3x^2 + 134a^4x + 120a^5$.
36. $531,441a^{12} - b^{12}$. 37. $-y^2z^2 - 3z^4$.
38. $3a^4b - a^3b^2 - 3a^2b^3 - 6a^2b^2 + 2ab^3 - 2ab^4 + 6b^4 - 6b^5$.
39. $-a^4 + 2a^2b^2 - b^4 + 2a^2c^2 + 2b^2c^2 - c^4$. 40. $b^2 - d^2$.
41. $4a^2 + 4b^2 + 4c^2 + 4d^2$. 46. $2q^2r$. 51. $-6x$.
42. $2a^2 + 2b^2 + 2c^2$. 47. 0. 52. $25a^4 - 25b^4$.
43. $ac - c^2$. 48. 0. 53. $24ab - 120b^2$.
44. $2mn$. 49. 0. 54. $3x^2y^4 + 2x^3y^3 - y^6$.
45. $2a^2 + 2b^2 + 2c^2$. 50. $-3bx$. 55. 0.

Exercise 15. Page 38.

- | | | |
|----------------------------------|--|-------------------------|
| 7. $x^2 + 2xy + y^2$. | 15. $a^2b^2 + 2abcd + c^2d^2$. | |
| 8. $y^3 - 2yz + z^2$. | 16. $9m^2n^2 - 24mn + 16$. | |
| 9. $4x^2 + 4x + 1$. | 17. $144 + 120x + 25x^2$. | |
| 10. $4a^2 + 20ab + 25b^2$. | 18. $16x^2y^4 - 8xy^3z^2 + y^2z^4$. | |
| 11. $1 - 2x^2 + x^4$. | 19. $9a^2b^2c^2 - 6ab^2c^2d + b^2c^2d^2$. | |
| 12. $9a^2x^3 - 24ax^3 + 16x^4$. | 20. $16x^6 - 8x^4y^2 + x^2y^4$. | |
| 13. $1 - 14a + 49a^2$. | 21. $x^2 - y^2$. | |
| 14. $25x^2y^2 + 20xy + 4$. | 22. $4a^2 - b^2$. | |
| 23. $9 - x^2$. | 26. $a^3x^4 - b^2y^3$. | 29. $1 - 9a^2b^6$. |
| 24. $9a^2b^2 - 4b^4$. | 27. $36x^2y^3 - 25y^4$. | 30. $a^4x^4 - b^4y^4$. |
| 25. $16x^4 - 9y^4$. | 28. $16x^{10} - 1$. | |

Exercise 16. Page 40.

- $x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$.
- $x^2 + y^2 + z^2 - 2xy + 2xz - 2yz$.
- $m^2 + n^2 + p^2 + q^2 + 2mn - 2mp - 2mq - 2np - 2nq + 2pq$.
- $x^4 + 4x^3 - 2x^2 - 12x + 9$.
- $x^4 - 12x^3 + 50x^2 - 84x + 49$.
- $4x^4 - 28x^3 + 85x^2 - 126x + 81$.
- $x^4 + y^4 + z^4 + 2x^2y^2 - 2x^2z^2 - 2y^2z^2$.
- $x^8 + 18x^4y^4 + y^8 - 8x^6y^2 - 8x^2y^6$.
- $a^6 + b^6 + c^6 + 2a^3b^3 + 2a^3c^3 + 2b^3c^3$.
- $x^6 + y^6 + z^6 - 2x^3y^3 - 2x^3z^3 + 2y^3z^3$.
- $x^2 + 4y^2 + 9z^2 + 4xy - 6xz - 12yz$.
- $x^4 + 4y^4 + 25z^4 - 4x^2y^2 + 10x^2z^2 - 20y^2z^2$.
- $x^4 + 4x^3 - 8x + 4$.
- $x^4 - 10x^3 + 39x^2 - 70x + 49$.
- $4x^4 - 12x^3 - 7x^2 + 24x + 16$.
- $x^2 + 4y^2 + 9z^2 + 4xy + 6xz + 12yz$.

Exercise 17. Page 42.

- | | | |
|---------------------|----------------------|----------------------|
| 1. $x^2 + 5x + 6$. | 3. $x^2 - 9x + 18$. | 5. $x^2 - 7x - 8$. |
| 2. $x^2 + 6x + 5$. | 4. $x^2 - 9x + 8$. | 6. $x^2 + 3x - 10$. |

- | | |
|---------------------------|-------------------------------|
| 7. $x^2 + 4x - 21$. | 14. $x^4 + 3x^2y^2 + 2y^4$. |
| 8. $x^2 - 6x + 8$. | 15. $x^4 - 2x^3y - 3x^2y^2$. |
| 9. $x^2 + 12x + 11$. | 16. $a^2x^2 - 3ax - 54$. |
| 10. $x^2 + ax - 6a^2$. | 17. $x^2 + (a-b)x - ab$. |
| 11. $x^2 - (c+d)x + cd$. | 18. $x^2 - 7x - 44$. |
| 12. $x^2 - 3xy - 4y^2$. | 19. $x^2 + x - 132$. |
| 13. $a^2 - 7ab + 10b^2$. | 20. $x^2 - 15x + 50$. |

Exercise 18. Page 43.

- | | |
|---|--|
| 1. $x^3 + 3x^2a + 3xa^2 + a^3$. | 5. $x^4 + 4x^3a + 6x^2a^2 + 4xa^3 + a^4$. |
| 2. $x^3 - 3x^2a + 3xa^2 - a^3$. | 6. $x^4 - 4x^3a + 6x^2a^2 - 4xa^3 + a^4$. |
| 3. $x^3 + 3x^2 + 3x + 1$. | 7. $x^4 + 4x^3 + 6x^2 + 4x + 1$. |
| 4. $x^3 - 3x^2 + 3x - 1$. | 8. $x^4 - 4x^3 + 6x^2 - 4x + 1$. |
| 9. $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$. | |
| 10. $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$. | |
| 11. $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$. | |
| 12. $x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$. | |

Exercise 19. Page 45.

- | | | | |
|----------|-----------|----------------------|---------------------------|
| 1. 66. | 4. -214. | 7. -11. | 10. $-0.022\frac{7}{8}$. |
| 2. 581. | 5. -21.7. | 8. $43\frac{3}{7}$. | 11. $0.3183+$. |
| 3. -128. | 6. -1.23. | 9. -0.1123. | 12. $0.0101321+$. |

Exercise 20. Page 47.

- | | | | |
|-----------------------|--------------------------|------------------------------|----------------------|
| 5. $3m$. | 11. $\frac{x}{5y}$. | 17. $\frac{9m}{y}$. | 23. $8ab^2z^2$. |
| 6. $-4a^3$. | 12. $-9a^4$. | 18. $-\frac{6y^2}{x}$. | 24. $-14x^4y^4z^6$. |
| 7. $\frac{5a}{c}$. | 13. $\frac{-3bm}{4ax}$. | 19. $\frac{4mx^4}{5a^3}$. | 25. $8bx^3$. |
| 8. $-\frac{1}{x^2}$. | 14. bc^2 . | 20. $6x^2z$. | 26. a^3bx . |
| 9. $6a$. | 15. m^4x^2 . | 21. $\frac{3bc^3d^2}{a^2}$. | 27. $17a^3$. |
| 10. $7ac$. | 16. $-17ay$. | 22. $\frac{3am^3n}{pq^3}$. | 28. $7a^{n-6}$. |

Exercise 21. Page 48.

3. $-2am + 3bn - 4cp$. 9. $-4xy^2 + 5x^2y + 8x^3$.
 4. $-7a + 6b - 5c$. 10. $x^3y^2 - 2x^2 + 3xy - 1$.
 5. $3x^4 - 2x^2 + 1$. 11. $3 - 2ab - a^2b^2$.
 6. $x - 2x^3 + 3x^5 - 4x^7$. 12. $-z - 2xz^2 + 5x^2yz^2 - 6x^3y^2$.
 7. $-5m^2 - 4my + 2y^2$. 13. $4ac^3 - 2a^2c^2 + 3a^3bc$.
 8. $2a^2 - 3ab + 6b^2$.

Exercise 22. Page 50.

6. $x - 4$. 8. $x^2 + x + 3$. 10. $3x^2 + 2x + 1$.
 7. $x - 8$. 9. $3x^2 - 2x + 4$. 11. $x^2 - 3x + 7$.
 12. $x^5 + x^4 + x^3 + x^2 + x + 1$. 14. $x^3 + 3x^2y + 9xy^2 + 27y^3$.
 13. $a^2 + ab - b^2$. 15. $x^4 + x^3y + x^2y^2 + xy^3 + y^4$.
 16. $a^4 - 2a^3b + 4a^2b^2 - 8ab^3 + 16b^4$.
 17. $2a^3 - 6a^2b + 18ab^2 - 27b^3$.
 18. $x^2 - 2x + 2$. 20. $x^2 - 5x + 6$. 22. $x^2 - x - 19$.
 19. $x^2 - 3x - 1$. 21. $x^2 - 4x + 8$. 23. $1 - 3x + 2x^2 - x^3$.
 24. $x^4 + 2x^3 + 3x^2 + 2x + 1$. 37. $4x^2 + 2xy + y^2$.
 25. $a^2 + 2ab + 3b^2$. 38. $-8a^3 + 2a^2b - ab^2$.
 26. $2x^3 - 3x^2 + 2x$. 39. $1 + x - 2x^2$.
 27. $a^4 + 3a^3 + 9a^2 + 27a + 81$. 40. $-1 - 3ab - 13a^2b^2$.
 28. $6x^2 - 7x + 8$. 41. $x^4 + 2x^3y + 2x^2y^2 + xy^3$.
 29. $x^2 - 3x - y$. 42. $x^3 - 3x^2y + 3xy^2 - y^3$.
 30. $x^2 - 3xy - 2y^2$. 43. $a^4 + a^2b^2 + 3b^4$.
 31. $x^2 + xy + y^2$. 44. $27x^2y - 18xy^2 - 9y^3$.
 32. $x^2 + xy - y^2$. 45. $a^2 + 2ab + 4b^2$.
 33. $x - y - z$. 46. $x^4 + 3x^3y + 8x^2y^2 - 8y^4$.
 34. $4 - 6x + 8x^2 - 10x^3$. 47. $4a^2 + 4ab + 3b^2$.
 35. $x + y$. 48. $a^2 + b^2 + c^2 - ab - ac - bc$.
 36. $x^2 - 3xy - y^2$. 49. $a + c + 2b$.
 50. $a^2 + 2ab + b^2 - ac - bc + c^2$.

Exercise 23. Page 53.

1. $(b+c)a-bc$.
2. $x-c$.
3. $x^2-(a+b)x+ab$.
4. $(x+a)x+b$.
5. $y^2-(m+n)y+mn$.
6. $(x+a)x-b$.
7. $x^2-(b+d)x+bd$.
8. $(x+c)x+d$.
9. $x^4+(1-m)x^3+(1-m+n)x^2+(1-m)x+1$.
10. $x+y+z$.

Exercise 24. Page 55.

1. y^2+y+1 .
2. $b^2+5b+25$.
3. $a^2+6a+36$.
4. $x^2+7x+49$.
5. $x^4+x^3y+x^2y^2+xy^3+y^4$.
6. $a^4+a^3+a^2+a+1$.
7. $1+2x+4x^2$.
8. $x^4+2x^3b+4x^2b^2+8xb^3+16b^4$.
9. $4a^2x^2+2ax+1$.
10. $1+3xy+9x^2y^2$.
11. $16a^3b^3+12abx+9x^2$.
12. $81a^4+27a^3+9a^2+3a+1$.
13. $16a^4+24a^3b+36a^2b^2+54ab^3+81b^4$.

Exercise 25. Page 56.

1. x^2-xy+y^2 .
2. $x^4-x^3y+x^2y^2-xy^3+y^4$.
3. $1-2a+4a^2$.
4. $9a^2-3ab+b^2$.
5. $4a^2x^2-2ax+1$.
6. $x^2-3xy+9y^2$.
7. $a^4-2a^3b+4a^2b^2-8ab^3+16b^4$.
8. $64x^2y^2-8xyz+z^2$.
9. $81a^2-54ab+36b^2$.
10. $16a^2-40ab+100b^2$.
11. $16a^3b^2-12abx+9x^2$.
12. $x^2-7x+49$.
13. $9x^2y^2-6xyz+4z^2$.
14. $256a^4-192a^3b+144a^2b^2-108ab^3+81b^4$.

Exercise 26. Page 57.

1. $x^3+x^2y+xy^2+y^3$.
2. $x^3-x^2y+xy^2-y^3$.
3. $a^5+a^4x+a^3x^2+a^2x^3+ax^4+x^5$.
4. $a^5-a^4x+a^3x^2-a^2x^3+ax^4-x^5$.
5. $x^3+3x^2y+9xy^2+27y^3$.
6. $x^3-3x^2y+9xy^2-27y^3$.
7. $8x^3+4x^2+2x+1$.
8. $8x^3-4x^2+2x-1$.
9. $27a^3x^3+9a^2x^2+3ax+1$.
10. $27a^3x^3-9a^2x^2+3ax-1$.

11. $32a^5 + 16a^4b + 8a^3b^2 + 4a^2b^3 + 2ab^4 + b^5$.
12. $32a^5 - 16a^4b + 8a^3b^2 - 4a^2b^3 + 2ab^4 - b^5$.
13. $x^5 + 3x^4y + 9x^3y^2 + 27x^2y^3 + 81xy^4 + 243y^5$.
14. $x^5 - 3x^4y + 9x^3y^2 - 27x^2y^3 + 81xy^4 - 243y^5$.
15. $27a^3 + 18a^2c + 12ac^2 + 8c^3$.
16. $27a^3 - 18a^2c + 12ac^2 - 8c^3$.
17. $64a^3 + 160a^2 + 400a + 1000$.
18. $64a^3 - 160a^2 + 400a - 1000$.
19. $125x^3 + 25x^2 + 5x + 1$.

Exercise 27. Page 58.

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|---|------------------------------|
| 1. $x^4 - x^2y^2 + y^4$. | 6. $x^3 - x^4 + 1$. |
| 2. $a^4 - a^2 + 1$. | 7. $16x^4 - 4x^2y^2 + y^4$. |
| 3. $a^8 - a^6y^2 + a^4y^4 - a^2y^6 + y^8$. | 8. $16 - 4a^2 + a^4$. |
| 4. $b^8 - b^6 + b^4 - b^2 + 1$. | 9. $81a^4 - 9a^2b^2 + b^4$. |
| 5. $a^8 - a^4b^4 + b^8$. | 10. $81c^4 - 9c^2 + 1$. |

Exercise 28. Page 61.

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|------------|--------------|--------------|--------------|
| 1. $x=4$. | 7. $x=6$. | 13. $x=-7$. | 19. $x=4$. |
| 2. $x=2$. | 8. $x=9$. | 14. $x=3$. | 20. $x=10$. |
| 3. $x=2$. | 9. $x=7$. | 15. $x=7$. | 21. $x=3$. |
| 4. $x=5$. | 10. $x=11$. | 16. $x=0$. | 22. $x=15$. |
| 5. $x=1$. | 11. $x=9$. | 17. $x=8$. | 23. $x=2$. |
| 6. $x=1$. | 12. $x=2$. | 18. $x=6$. | 24. $x=3$. |
| | | | 25. $x=4$. |

Exercise 29. Page 62.

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|--|--------|--------|-----------|----------------------------|
| 6. 70. | 7. 43. | 8. 23. | 9. 21, 7. | 10. 36, 26, 18, 12. |
| 11. 8, 12. | | | | 14. \$68. |
| 12. Father, 50 yrs.; son, 30 yrs. | | | | 15. Cloth, \$3; silk, \$6. |
| 13. 30, 26, 22, 18, 14, 10. | | | | 16. 52. |
| 17. A, \$130; B, \$150; C, \$130; D, \$90. | | | | |
| 18. 152 men; 76 women; 38 children. | | | | |
| 19. 13, 21. | | | | 20. A, 84 yrs.; B, 26 yrs. |

21. Wife, \$4000; each son, \$1000; each daughter, \$500.
 22. 49 gallons. 24. 50, 40. 26. 24, 60.
 23. \$21. 25. 42, 18. 27. A, 88 years; B, 44 years.
 28. 18 years. 30. A, 57 years; B, 19 years.
 29. 40 years. 31. 4 years.
 32. 10 dollar-pieces; 40 twenty-five-cent pieces.
 33. 28 pounds of better kind; 2 pounds of poorer kind.
 34. 28 days. 35. 60 gallons @ \$2; 40 gallons @ \$3.
 36. 40 children. 37. $31\frac{1}{2}$ gallons. 38. \$45.
 39. 2 half-dollars; 6 quarter-dollars; 24 dimes; 48 half-dimes.
 40. 10 half-dollars; 41 quarter-dollars.
 41. 80 guineas; 128 half-crowns.

Exercise 30. Page 68.

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|--------------------------|-------------------------------------|
| 1. $5a(a-3)$. | 6. $3a^3b^2(2a^2b-7a+9b^2)$. |
| 2. $6a(a^2+3a-2)$. | 7. $27x^2y^8(2+4x^2y^2-9x^4y^3)$. |
| 3. $7(7x^2-3x+2)$. | 8. $45x^4y^7(x^3y^3-2x-8y)$. |
| 4. $4xy(x^2-3xy+2y^2)$. | 9. $70ay^4(a^2-2ay+3y^2)$. |
| 5. $y(y^3-ay^2+by+c)$. | 10. $32a^3b^6(1+3a^3b^2-4a^5b^3)$. |

Exercise 31. Page 69.

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|---------------------|-----------------------|
| 1. $(x-a)(x-b)$. | 6. $(ab+pq)(x-y)$. |
| 2. $(a-y)(b+y)$. | 7. $(cx+ay)(dx-by)$. |
| 3. $(b-x)(c+x)$. | 8. $(ac-bd)(by-dx)$. |
| 4. $(m+a)(x+n)$. | 9. $(a-b)(x-y)$. |
| 5. $(cx+y)(dx-y)$. | 10. $(cz+y)(dz-y)$. |

Exercise 32. Page 71.

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|--------------------|---------------------|
| 1. $(x+8)(x+3)$. | 5. $(x+11)(x+10)$. |
| 2. $(x+6)(x+5)$. | 6. $(y+20)(y+15)$. |
| 3. $(y+12)(y+5)$. | 7. $(b+17)(b+6)$. |
| 4. $(z+12)(z+1)$. | 8. $(x+2)(x+1)$. |

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|-----------------------------------|------------------------------|
| 9. $(x+6)(x+1)$. | 17. $(x^4y^2+4)(x^4y^2+3)$. |
| 10. $(a+8b)(a+b)$. | 18. $(z^5+8)(z^5+2)$. |
| 11. $(x+9a)(x+4a)$. | 19. $(a+5b)(a+4b)$. |
| 12. $(y+16p)(y+3p)$. | 20. $(x^3+5)(x^3+4)$. |
| 13. $(z+25q)(z+4q)$. | 21. $(ax+11b)(ax+3b)$. |
| 14. $(a^2+3)(a^2+2)$. | 22. $(ac+5x)(ac+2x)$. |
| 15. $(z^3+3)(z^3+1)$. | 23. $(xyz+16)(xyz+3)$. |
| 16. $(ab+16)(ab+2)$. | 24. $(bc+13a)(bc+5a)$. |
| 25. $(rs+18z)(rs+5z)$. | |
| 26. $(m^2n^2+17pq)(m^2n^2+3pq)$. | |

Exercise 33. Page 72.

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|------------------------------|----------------------------|
| 1. $(x-5)(x-2)$. | 13. $(abc-11)(abc-2)$. |
| 2. $(x-19)(x-10)$. | 14. $(x-10)(x-5)$. |
| 3. $(a-12)(a-11)$. | 15. $(x-10)(x-10)$. |
| 4. $(b-20)(b-10)$. | 16. $(ax-18)(ax-3)$. |
| 5. $(z-23)(z-20)$. | 17. $(ax-13b)(ax-3b)$. |
| 6. $(x-6)(x-1)$. | 18. $(ac-13z)(ac-11z)$. |
| 7. $(x^2-3a^2)(x^2-a^2)$. | 19. $(x-13)(x-7)$. |
| 8. $(x-6)(x-2)$. | 20. $(x-15)(x-8)$. |
| 9. $(z-56)(z-1)$. | 21. $(z-45)(z-8)$. |
| 10. $(y^3-4)(y^3-3)$. | 22. $(x-a)(x-c)$. |
| 11. $(xy-26)(xy-1)$. | 23. $(yz-17ab)(yz-11ab)$. |
| 12. $(a^2b^3-6)(a^2b^3-5)$. | 24. $(cd-17ab)(cd-13ab)$. |

Exercise 34. Page 73.

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|---------------------|-------------------------|
| 1. $(x+7)(x-1)$. | 8. $(a+30)(a-5)$. |
| 2. $(x+12)(x-7)$. | 9. $(b^4+4)(b^4-1)$. |
| 3. $(y+12)(y-5)$. | 10. $(bc+14)(bc-11)$. |
| 4. $(y+15)(y-3)$. | 11. $(c^5+20)(c^5-5)$. |
| 5. $(z+12)(z-1)$. | 12. $(c+30)(c-13)$. |
| 6. $(z+20)(z-7)$. | 13. $(a+12)(a-11)$. |
| 7. $(a+25)(a-12)$. | 14. $(xyz+11)(xyz-2)$. |

Exercise 35. Page 73.

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|------------------------|-------------------------------|
| 1. $(x-7)(x+4)$. | 9. $(y-10a)(y+5a)$. |
| 2. $(y-9)(y+2)$. | 10. $(ab-4)(ab+1)$. |
| 3. $(x-12)(x+3)$. | 11. $(ax-9)(ax+6)$. |
| 4. $(z-15)(z+4)$. | 12. $(cd-30)(cd+6)$. |
| 5. $(z-14)(z+1)$. | 13. $(a^3c-2)(a^3c+1)$. |
| 6. $(a-20)(a+5)$. | 14. $(y^4z^2-12)(y^4z^2+7)$. |
| 7. $(c^5-10)(c^5+1)$. | 15. $(ab-18)(ab+2)$. |
| 8. $(x-10)(x+2)$. | 16. $(x-a)(x+b)$. |

Exercise 36. Page 74.

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|-------------------|-------------------------|----------------------|
| 1. $(x+6)^2$. | 5. $(y+100)^2$. | 9. $(y^3+12)^2$. |
| 2. $(x+14)^2$. | 6. $(z^2+7)^2$. | 10. $(xz+81)^2$. |
| 3. $(x+17)^2$. | 7. $(x+18y)^2$. | 11. $(2a+3b^2)^2$. |
| 4. $(z+1)^2$. | 8. $(y^2+8z^2)^2$. | 12. $(3xy^2+5z)^2$. |
| 13. $(3x+2y)^2$. | 14. $(2a^2x+5x^2y)^2$. | |

Exercise 37. Page 75.

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|----------------------|-------------------------|----------------------------|
| 1. $(a-4)^2$. | 9. $(z^3-17)^2$. | 17. $(3mn-4)^2$. |
| 2. $(a-15)^2$. | 10. $(2x^2y-5y^2z)^2$. | 18. $(2bx-3xy)^2$. |
| 3. $(x-19)^2$. | 11. $(4xy^2-yz^2)^2$. | 19. $(7a-8b)^2$. |
| 4. $(x-20)^2$. | 12. $(3abc-bcd)^2$. | 20. $(8x^2y^3-10x^2z)^2$. |
| 5. $(y-50)^2$. | 13. $(4x^3-xy^2)^2$. | 21. $(7abc-2x)^2$. |
| 6. $(y^2-10)^2$. | 14. $(a^3x^2-by^4)^2$. | 22. $(11x^2-13y)^2$. |
| 7. $(y-25z)^2$. | 15. $(6xy-5y^2)^2$. | 23. $(17xyz-3yzd)^2$. |
| 8. $(x^2-16y^2)^2$. | 16. $(1-3ab^3)^2$. | 24. $(19xyz-2abc)^2$. |

Exercise 38. Page 77.

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|----------------------------|-------------------------------------|
| 1. $(a+b)(a-b)$. | 5. $(a^2+1)(a+1)(a-1)$. |
| 2. $(a+4)(a-4)$. | 6. $(a^4+b^4)(a^2+b^2)(a+b)(a-b)$. |
| 3. $(2a+5)(2a-5)$. | 7. $(a^4+1)(a^2+1)(a+1)(a-1)$. |
| 4. $(a^2+b^2)(a+b)(a-b)$. | 8. $(6x+7y)(6x-7y)$. |

9. $(10xy + 11ab)(10xy - 11ab)$.
 10. $(1 + 7x)(1 - 7x)$. 12. $(a - b + c)(a - b - c)$.
 11. $(a^2 + 5b)(a^2 - 5b)$. 13. $(x + a - b)(x - a + b)$.
 14. $(a + b + c + d)(a + b - c - d)$.
 15. $4xy$. 17. $(x + y + z)(x - y - z)$.
 16. $(1 + a - b)(1 - a + b)$. 18. $(x - y + z)(x - y - z)$.
 19. $(a + 2b - 3c)(a - 2b + 3c)$.
 20. $(a - y + x + z)(a - y - x - z)$.
 21. $(z + x - y)(z - x + y)$.
 22. $(x - y + z + d)(x - y - z - d)$.
 23. $(x - z + y - a)(x - z - y + a)$.
 24. $(a + b + c)(a + b - c)$.
 25. $(a - b + x - y)(a - b - x + y)$.
 26. $(ax + by + 1)(ax + by - 1)$.
 27. $(1 + x - y)(1 - x + y)$. 30. $(x + y + 2)(x - y)$.
 28. $12(a - 1)(2a + 1)$. 31. $(x + y)(x - y + 2)$.
 29. $(a - b + x)(a - b - x)$. 32. $(d + x - 2y)(d - x + 2y)$.
 33. $(a + b + c)(a - b - c)$.
 34. $(2x^2 + 3x - 1)(2x^2 - 3x + 1)$.

Exercise 39. Page 78.

1. $(a - b)(a^2 + ab + b^2)$. 3. $(x - 7)(x^2 + 7x + 49)$.
 2. $(x - 2)(x^2 + 2x + 4)$. 4. $(y - 5)(y^2 + 5y + 25)$.
 5. $(y - 6)(y^2 + 6y + 36)$.
 6. $(2x - 3y)(4x^2 + 6xy + 9y^2)$.
 7. $(4y - 10z)(16y^2 + 40yz + 100z^2)$.
 8. $(9x - 8y)(81x^2 + 72xy + 64y^2)$.
 9. $(3a - 12)(9a^2 + 36a + 144)$.
 10. $(10a - 11b)(100a^2 + 110ab + 121b^2)$.

Exercise 40. Page 79.

1. $(x + y)(x^2 - xy + y^2)$. 3. $(x + 6)(x^2 - 6x + 36)$.
 2. $(x + 2)(x^2 - 2x + 4)$. 4. $(y + 4z)(y^2 - 4yz + 16z^2)$.

5. $(4b + 5c)(16b^2 - 20bc + 25c^2)$.
6. $(6a + 8c)(36a^2 - 48ac + 64c^2)$.
7. $(9x + 12y)(81x^2 - 108xy + 144y^2)$.
8. $(x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$.
9. $(x + y)(x^6 - x^5y + x^4y^2 - x^3y^3 + x^2y^4 - xy^5 + y^6)$.
10. $(2b + 3c)(16b^4 - 24b^3c + 36b^2c^2 - 54bc^3 + 81c^4)$.

Exercise 41. Page 79.

1. $(a^2 + b^2)(a^4 - a^2b^2 + b^4)$.
2. $(a^2 + b^2)(a^8 - a^6b^2 + a^4b^4 - a^2b^6 + b^8)$.
3. $(x^4 + y^4)(x^8 - x^4y^4 + y^8)$.
4. $(b^2 + 4c^2)(b^4 - 4b^2c^2 + 16c^4)$.
5. $(x^2 + 1)(x^4 - x^2 + 1)$.
7. $(4a^2 + x^2)(16a^4 - 4a^2x^2 + x^4)$.
6. $(a^4 + 1)(a^8 - a^4 + 1)$.
8. $(9 + c^2)(81 - 9c^2 + c^4)$.

Exercise 42. Page 80.

1. $(a^2 + ab + b^2)(a^2 - ab + b^2)$.
2. $(3x^2 + 3xy + 2y^2)(3x^2 - 3xy + 2y^2)$.
3. $(4x^2 + 3xy - y^2)(4x^2 - 3xy - y^2)$.
4. $(9a^2 + 7ab + 4b^2)(9a^2 - 7ab + 4b^2)$.
5. $(9a^2 + 10ab + 4b^2)(9a^2 - 10ab + 4b^2)$.
6. $(3x^2 + 2xy + 7y^2)(3x^2 - 2xy + 7y^2)$.
7. $(5a^2 + 7ab + 4b^2)(5a^2 - 7ab + 4b^2)$.
8. $(7m^2 + 4mn + 9n^2)(7m^2 - 4mn + 9n^2)$.
9. $(3a^2 + 3ac + 5c^2)(3a^2 - 3ac + 5c^2)$.
10. $(7a^2 + 13ab + 11b^2)(7a^2 - 13ab + 11b^2)$.
11. $(8x^2 + 4xy + 9y^2)(8x^2 - 4xy + 9y^2)$.
12. $(2x^2 + 5xy - 3y^2)(2x^2 - 5xy - 3y^2)$.
13. $(5x^2 + xy - 4y^2)(5x^2 - xy - 4y^2)$.
14. $(9x^2 + 4xy - y^2)(9x^2 - 4xy - y^2)$.

Exercise 43. Page 81.

1. $(4x + 1)(3x - 2)$.
3. $(4x + 1)(3x - 1)$.
2. $(3x - 1)(4x - 1)$.
4. $(x + 1)(3x - 5)$.

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|------------------------------|------------------------|
| 5. $(x+2)(3x-2)$. | 15. $(2x+1)(2x+3)$. |
| 6. $(3x+4)(2x-1)$. | 16. $(a+2x)(a-3x)$. |
| 7. $(4x+1)(x+3)$. | 17. $(2a+5b)(4a-3b)$. |
| 8. $(x+3)(4x-1)$. | 18. $(3a-2c)(2a-5c)$. |
| 9. $(2x+1)(2x-3)$. | 19. $(4x+3y)(2x+7y)$. |
| 10. $(x-a)(x-2a)$. | 20. $(4x+3y)(2x-7y)$. |
| 11. $(3a^2+x^2)(4a^2-x^2)$. | 21. $(2x+7y)(3x-y)$. |
| 12. $(2x+y)(x+2y)$. | 22. $(11a-b)(a-2b)$. |
| 13. $(2ax+1)(3ax-1)$. | 23. $(2c-d)(c-6d)$. |
| 14. $(3b+x)(2b-3x)$. | 24. $(2y+3z)(3y-z)$. |

Exercise 44. Page 82.

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|----------------|----------------|------------------|
| 1. $(a+b)^3$. | 4. $(x+y)^4$. | 7. $(x+y+z)^2$. |
| 2. $(a+1)^3$. | 5. $(x-1)^4$. | 8. $(x-y-z)^2$. |
| 3. $(a-1)^3$. | 6. $(a-c)^4$. | 9. $(a+b-c)^2$. |

Exercise 45. Page 84.

- $(2x-y-3)(x-2y-7)$.
- $(6x-y+1)(x-6y-1)$.
- $(3x+2y+1)(2x-3y-1)$.
- $(5x-3y+2)(x-y+1)$.
- $(x+y-3)(2x-3y-2)$.
- $(x+5y-3)(x-5y-7)$.
- $(2x-y+z)(x-2y-z)$.
- $(2x+y-3z)(3x-y+3z)$.
- $(6x-y-z)(x-y+6z)$.
- $(5x-3y+2z)(x-y-z)$.
- $(x+y+z)(2x-3y-2z)$.
- $(2x-3y+2z)(3x-2y+3z)$.
- $(x-y)(x-y+5)$.
- $(2x-y)(x+3y-2z)$.

Exercise 46. Page 85.

1. $5(x+1)(x-4)$.
2. $2x^3(x-2)(x-6)$.
3. $3(ab+1)(ab-4)$.
4. $(a+x)(a+x+4)$.
5. $(a-b+c)(a-b-c)$.
6. $(x-y+c-d)(x-y-c+d)$.
7. $(2+x+x^2)(2-x-x^2)$.
8. $(a+b)(a-b-1)$.
9. $(a^2+a+1)(a^2-a+1)$.
10. $(x-y)(x+y-z)$.
11. $(a-b)(b-c)$.
12. $(3x-y)(x-z)$.
13. $(a+x)(a-x-b)$.
14. $(a-x)(a-x+1)$.
15. $(x-y)(3x+3y-2)$.
16. $x(x^2+1)(x+1)$.
17. $(ax-1)(a^3x^3-ax-1)$.
18. $(x+3y)(x-3y)(3x-2y)$.
19. $(2x-1)(x+1)(2x^2-x+1)$.
20. $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$.
21. $(x^2+y^2)(x^4-x^2y^2+y^4)$.
22. $(3+x)(3-x)(9+3x+x^2)(9-3x+x^2)$.
23. $y(x^4+y^4)(x^8-x^4y^4+y^8)$.
24. $c(a^2+c^2)(a+c)(a-c)$.
25. $(x+7)(x-3)$.
26. $3(a-2b)(a-5b)$.
27. $2x^2(x+y)(x-3y)$.
28. $(2a-b)^2$.
29. $4(2x-5y)^2$.
30. $x^2y^2(6a+5bx)(6a-5bx)$.
31. $(3xy^2-5z)^2$.
32. $x(4x^2+1)(2x+1)(2x-1)$.
33. $(x-y-z)^2$.
34. $(a-3b)(a+2b-4)$.
35. $(x+y+2)(x+y-3)$.
36. $(a^2+2ab+b^2+c^2)(a+b+c)(a+b-c)$.
37. $(x-3y)(x+2y-4)$.
38. $(1+x^2)(1-x)$.
39. $(3x-2y)(x-3y)$.
40. $(x+13)(x+7)$.
41. $(x-y)(x-z)(z-y)$.
42. $(x+3)(x-8)$.
43. $(x+y-z)(x-y+z)(x+y+z)(x-y-z)$.
44. $5x(xy+4z)(xy-3z)$.
45. $(x^2+1)(3x-1)$.
46. $(x-m+n)(x-m-n)$.
47. $(a+b+c)(a+b-c)(c+a-b)(c-a+b)$.

48. $a^5(a^2+1)$. 50. $(y+9)(y-13)$.
 49. $(1-7a^3x)^2$. 51. $(x+15)(x-9)$.
 52. $(2a-3b+2c)(2a-3b-2c)$.
 53. $(a+3c)(a-3c+6b)$.
 54. $(3x+2y-z)(3x-2y+z)$.
 55. $x^2(3b+x)(2b-3x)$.
 56. $(a-b)^3$. 59. $3y^2(x-y)(3x+2y)$.
 57. $(x+y)^3$. 60. $(3x+2y)(2x+3y)$.
 58. $ab(a-b)$. 61. $b^2(3a+4b)(2a-3b)$.
 62. $(a+2b-3c+d)(a-2b+3c+d)$.
 63. $(x^2+4y^2)(x-2y)$. 68. $(x+y-z)(x-y+z+1)$.
 64. $(2ax-b)(2ax-3b)$. 69. $2(x+y)(x+y+a)$.
 65. $(3x-2y)(6x-4y+3)$. 70. $4b(2a+3c)(2a+c)$.
 66. $2(x+3y)(x-2y+3z)$. 71. $(m+n)(m-n)(p-q)$.
 67. $(x+y+1)(1-x)(y-1)$. 72. $2a(3x+y)(2x-3y)$.
 73. $2x(x+7)(x-5)$.
 74. $2x(2a-x)(4a^2+2ax+x^2)$.
 75. $4b(2x-y)(4x^2+2xy+y^2)$.
 76. $x(1-3x)(1+3x+9x^2)$.
 77. $(x^2+y^2)(x+y)(x-y)(x^4-x^2y^2+y^4)$
 $(x^2-xy+y^2)(x^2+xy+y^2)$.
 78. $(7m+11n)(7m-11n)$. 80. $(3z^2+2)(4z^2-3)$.
 79. $(4+9y^2)(2+3y)(2-3y)$. 81. $(x^2+1)(x-1)$.
 82. $(x+y+1)(x-y+1)$.
 83. $(7a-7b+8m-8n)(7a-7b-8m+8n)$.
 84. $\left\{ \frac{(a+b+c)(a+b-c)}{2ab} \right\} \left\{ \frac{(c+a-b)(c-a+b)}{2ab} \right\}$.
 85. $(x-8)(x-45)$. 87. $(2b-e)(a+b-c)$.
 86. $(x+2)(x-2)(x-2y+1)$. 88. $5x(5x^2+7y^2)^2$.
 89. $a(a+b)(a^2+ab+b^2)(a^2-ab+b^2)$.
 90. $2x(a+c)(a-c)(a^2-ac+c^2)$.
 91. $(3x+2y-3z)(2x-3y+3z)$.
 92. $(4x-y+z)(x-2y-z)$.

93. $(3a - b + 2c)(a - 2b + c)$.

94. $(x - 1)(x^3 - x^2 - 8)$.

95. $(5x - 3y)(x - y - 1)$.

96. $(a + 2b - 3c - d)(a - 2b + 3c - d)$.

97. $(x + 2)(x - 3)(x + 4)(x - 5)$.

Exercise 47. Page 89.

1. $18abcd$.

9. $7x - 4$.

17. $5y$.

2. $17pq$.

10. $2a^2xy(3x - y)$.

18. $x(x + 1)$.

3. $4x^2y^2z^2$.

11. $2abc$.

19. $3(x - 1)$.

4. $30x^2y^3$.

12. $x - 3$.

20. $2(a - b)$.

5. $a - b$.

13. $2(a + b)$.

21. $x + y$.

6. $a - x$.

14. $6x^2(x - y)$.

22. $x - y$.

7. $a + x$.

15. $3x(x + 4)$.

23. $x - 1$.

8. $3x + 1$.

16. $c(a - b)$.

Exercise 48. Page 96.

1. $5x - 1$.

9. $2x^2 - 3x + 3$.

17. $a - 2$.

2. $x + 1$.

10. $2x - 7$.

18. $2(2y + 5)$.

3. $a(3a - 1)$.

11. $5x(6x - 1)$.

19. $3x(2x^2 - 5)$.

4. $(3x + 2)(3x - 2)$.

12. $y(3x - 4y)$.

20. $x^3 - 5x + 1$.

5. $3x(3x^2 - x + 1)$.

13. $3x - 5$.

21. $7x + 1$.

6. $2(x - 2)$.

14. $4x + 1$.

22. $y(3x^2 - xy - 5y^2)$.

7. $2x - 5$.

15. $3(x + 3)$.

23. $2x^3 - 4x^2 + x - 1$.

8. $a(a - 2x)$.

16. $x^2 + xy + y^2$.

24. $x - 2a$.

Exercise 49. Page 97.

1. $x + 1$.

5. $y^2 - 2y + 5$.

9. $2(x + y)$.

2. $y - 1$.

6. $x - 1$.

10. $x^2 - xy + y^2$.

3. $x^2 - 2x + 5$.

7. $x^2 + x + 1$.

11. $xy(x - y)$.

4. $x - 2$.

8. $x + 1$.

Exercise 50. Page 98.

1. $12a^3x^2$.
2. $72ax^2y^2$.
3. $x^2(a+x)$.
4. $x(x+1)(x-1)$.
5. $a(a+b)(a-b)$.
6. $(2x+1)(2x-1)$.
7. a^3+b^3 .
8. $(x^2+1)(x+1)(x-1)$.
9. $x(x^3+1)(x^3-1)$.
10. $x(x+1)(x^3-1)$.
11. $(2a-1)(8a^3+1)$.
12. $(a-b)(a+b)^2$.
13. $4(1+x)(1-x)$.
14. x^3-1 .
15. $(x+y)^2(x-y)^2$.
16. $12(x-y)^2(x^3+y^3)$.
17. $120xy(x+y)(x-y)$.
18. $(x+2)(x+3)(x+4)$.
19. $(a-3)(a+4)(a-5)$.
20. $(x+5)(x+6)(x+7)$.
21. $(x+2)(x-2)(x-11)$.
22. $20a^2b(a-b)(a-2b)(a+3b)$.
23. $240(x+1)(x-1)(x+2)(x-2)$.
24. $12x^2y^2(x+y)^2(x-y)^2$.
25. $(a-b)(b-c)(c-a)$.
26. $(a-b)(a-c)(b-c)$.
27. $x^3(x-1)(x-3)(x+4)$.
28. $12xy(x-y)^3$.
29. $(a+b+c+d)(a+b-c-d)(a+c-b-d)(a+d-b-c)$.
30. $60(x-3)(x-2)^2$.

Exercise 51. Page 101.

1. $(2x+1)(3x-2)(7x-1)$.
2. $(x+1)(x-1)(x+3)(2x+1)(3x-2)$.
3. $(x-12)(x-3)(x^2-2)(x^2+3x+9)$.
4. $(5x-1)(2x+3)(x+4)$.
5. $2(3x-2y)(3x+5y)(4x+3y)$.
6. $2x(x^2+1)(x-1)(x^2-x-1)$.
7. $12(2x+1)(2x-1)(3x+2)(x-3)(x-4)$.
8. $(x-1)(x-2)(x-3)(x-4)$.
9. $(x+2a)(x-2a)(x^2+4a^2)$.
10. $(x+y)(x-y)(x+2y)(x-2y)$.
11. $1+p^2+p^4$.
12. $(1-a)^3$.

13. $(a+b+c)(a-b+c)(a+b-c)(b+c-a).$

14. $c(3c^2+y^2)(4c+3y)(c-y).$

15. $(m+3)(m^2-3m+1)(m^5+1).$

16. $(5n^2-1)(4n^2+1)(5n^2+n+1).$

17. $(b-1)(b^3-b^2-8)(4b^2-8b+1).$

18. $6r(r+1)^2(r-1)^4.$

Exercise 52. Page 105.

1. $\frac{x-1}{4x}.$

11. $\frac{a^2+5a+5}{a^2+a-2}.$

21. $\frac{a^2+b^2}{a}.$

2. $\frac{x-5}{x-3}.$

12. $\frac{3x-1}{x^2-1}.$

22. $\frac{a+b}{a-2b}.$

3. $\frac{x+1}{x-7}.$

13. $\frac{x^2-2x+2}{x^2-2}.$

23. $\frac{3(a+b)}{4ab(a-b)}.$

4. $x^2-x+1.$

14. $\frac{2x-3a}{4x^2+6ax+9a^2}.$

24. $\frac{a+b+c}{a}.$

5. $\frac{x^3+y^3}{x^3-y^3}.$

15. $\frac{5a+2b}{3a+2b}.$

25. $\frac{x(3x-y)}{y(3x+2y)}.$

6. $\frac{a^2-a+1}{a^2+a+1}.$

16. $\frac{a-b-c}{a+b-c}.$

26. $\frac{a+b+c+d}{a-b+c+d}.$

7. $\frac{a-5}{a-3}.$

17. $\frac{x^3+x^2-2}{2x^2+2x+1}.$

27. $\frac{3x+2}{4x+5}.$

8. $\frac{x-2}{x+4}.$

18. $\frac{x-3}{x+1}.$

28. $\frac{x^2-xy+y^2}{(x-y)^2}.$

9. $\frac{x-3}{x+1}.$

19. $\frac{x-3}{x-2}.$

29. $x^2+y^2.$

10. $\frac{x^2+xy-y^2}{x^2-xy-y^2}.$

20. $\frac{(x-1)^2}{x^2-3x+1}.$

30. $\frac{a+b}{a-b}.$

Exercise 53. Page 107.

1. $x - 1$.
2. $3x - 10 + \frac{41}{x+4}$.
3. $3x - 6 + \frac{29}{x+4}$.
4. $a - 2x + \frac{3x^2}{a+x}$.
5. $2x + 6 + \frac{23}{x-3}$.
6. $2a - 3x + \frac{7x^2}{5a-x}$.
7. $12x + 3 + \frac{19}{4x-1}$.
8. $2x + 3 + \frac{10}{x-4}$.
9. $a + b + \frac{2b^2}{a-b}$.
10. $x - 1 + \frac{5x+4}{5x^2+4x-1}$.

Exercise 54. Page 109.

1. $\frac{2y}{x+y}$.
2. $\frac{2x}{x+y}$.
3. $\frac{x^2-1}{x}$.
4. $\frac{2(a^2-ax+x^2)}{a-x}$.
5. $\frac{2(11a^2-20ab+8b^2)}{5a-6b}$.
6. $\frac{2ab}{a+b}$.
11. $\frac{x^2-2xy-y^2}{x+y}$.
16. $\frac{3x^2+2x+1}{x+4}$.
7. $\frac{2(19a-23a^2-1)}{5-6a}$.
12. $\frac{29a}{4}$.
17. $\frac{x^3+1}{x-1}$.
8. $\frac{ax+3}{2a}$.
13. $\frac{a^2}{a+1}$.
18. $\frac{x(x^3-2x^2-3)}{x-2}$.
9. $\frac{2a}{a-b}$.
14. $\frac{x^2}{x-3}$.
19. $\frac{a^3+2x^3}{a+2x}$.
10. $\frac{-2b}{a+b}$.
15. $\frac{2a^2-ab-b^2}{a+b}$.
20. $\frac{x^2+xy+y^2}{x+a}$.

Exercise 55. Page 111.

1. $\frac{9x-21}{18}, \frac{4x-9}{18}$.
2. $\frac{4x-8y}{10x^2}, \frac{3x^2-8xy}{10x^2}$.
3. $\frac{48a^2-60ac}{60a^2c}, \frac{15a-10c}{60a^2c}$.
4. $\frac{5+5x}{1-x^2}, \frac{6}{1-x^2}$.

5. $\frac{a-c}{(a-b)(a-c)(b-c)}, \frac{b-c}{(a-b)(a-c)(b-c)}.$
 6. $\frac{8x^2(a-b)}{6(a^2-b^2)}, \frac{xy}{6(a^2-b^2)}.$
 7. $\frac{30(4x+1)}{15(x-2)}, \frac{5(2x-1)}{15(x-2)}, \frac{3(3x+2)}{15(x-2)}.$
 8. $\frac{an-bmn}{mnx}, \frac{mnx}{mnx}, \frac{cm-bmn}{mnx}.$

Exercise 56. Page 113.

1. $\frac{16x^2+55x+4xy-55y}{50x}.$ 6. $\frac{11y^2-7x^2-4xy-8x^2y^2}{x^3y^3}.$
 2. $\frac{27x^2-2x^2y-16xy-28y^2}{12x^2}.$ 7. $\frac{b^2c^2-2ab^2c+2a^2bc-a^2c^2}{a^2b^2c^2}.$
 3. $\frac{180a^2+54ab-20ab^2+331b^2}{90b^2}.$ 8. $\frac{6a^5-3a^4-2}{8a^2}.$
 4. $\frac{80x^3+64x^2+84x+45}{60x^2}.$ 9. $\frac{a^2b+b^2c+ac^2}{abc}.$
 5. $\frac{32x+9y}{42}.$ 10. $-\frac{1}{6y^2z}.$

Exercise 57. Page 114.

1. $\frac{2x-1}{x^2-x-30}.$ 3. $\frac{2}{1-x^2}.$ 5. $\frac{2x-y}{(x-y)^2}.$
 2. $\frac{4}{x^2-10x+21}.$ 4. $-\frac{1}{1+x}.$ 6. $\frac{1}{a^2-x^2}.$
 7. $\frac{a^3-a^2b-ab^2-b^3}{ab(a^2-b^2)}.$
 8. $\frac{7x-17}{4x(x^2-3x+2)}.$ 9. $\frac{2x^3}{1+x^2+x^4}.$ 10. $\frac{2a-3b}{x^2-y^2}.$

Exercise 58. Page 115.

1. $\frac{2}{1-a}.$ 2. $\frac{4x}{1-x^4}.$ 3. $\frac{2x+x^3}{1+x^2}.$ 4. $\frac{x+y}{y}.$

5. $\frac{3x^3 - 24x^2 + 60x - 46}{x^3 - 9x^2 + 26x - 24}$. 6. $\frac{3x^2 - 2ax - 6a^2}{(x-a)^3}$.
7. $\frac{6}{(x^2-1)(x+2)}$. 8. 0. 9. 2. 10. $\frac{y}{x+y}$. 11. 0.
12. $\frac{2ab}{(a+c)(b+c)}$.
13. $\frac{ax}{(a-x)(a+2x)}$. 15. $-\frac{2(x^2 - xy + y^2)}{xy(x-y)}$.
14. $\frac{9a - 10b + c}{(a-b)(a-c)(b-c)}$. 16. $\frac{2(b-a)}{y(a+b)}$.
17. $\frac{2x^3 - 5x^2y + 10xy^2 + 5y^3}{(x+y)^2(x-y)^2}$.
18. $\frac{ac - ab + b^2 - c^2}{(a+b+c)(a+b-c)(a+c-b)}$. 19. $\frac{3ab(x-y)}{a^2x^2 - b^2y^2}$.

Exercise 59. Page 119.

1. $\frac{y}{x-y}$. 3. $\frac{3x^2}{x^2-1}$. 5. 0.
2. $\frac{1}{2+x}$. 4. $\frac{6+y}{3(1+y)(1-y)}$. 6. $\frac{1}{(x+a)(x+b)}$.
7. $\frac{a^6 + 2a^5b - 2ab^5 + b^6}{(a^3+b^3)(a^3-b^3)}$. 8. $\frac{b^2 - ax}{b^2 - x^2}$.
9. $\frac{1}{2+x}$. 10. 0. 11. 0. 12. 0. 13. 0.
14. $\frac{9a - 10b + c}{(a-b)(a-c)(b-c)}$. 15. $\frac{1}{z(z-x)(y-z)}$.

Exercise 60. Page 122.

1. $\frac{ac}{bd}$. 4. $\frac{4xy}{5b}$. 7. $-\frac{15x}{4z}$. 10. $\frac{a-b}{a^2}$.
2. $9ax$. 5. $\frac{by}{9ax}$. 8. $\frac{3mnxy}{4pq^2}$. 11. $\frac{a^2+b^2}{(a-b)^2}$.
3. $\frac{3}{4}$. 6. $-ax$. 9. $\frac{5km^2}{4pq}$. 12. $\frac{(x-1)(x-6)}{x^2}$.

13. $\frac{x-6}{x-3}$. 18. $\frac{x}{x^2+y^2}$. 23. $\frac{3a}{2b}$.
 14. $\frac{(a+x)(a^2+ax+x^2)}{(a-x)(a^2-ax+x^2)}$. 19. $-\frac{m+n}{c^2-cd+d^2}$. 24. $\frac{c-a+b}{c-a-b}$.
 15. $\frac{2ax^2(x-y)}{c}$. 20. 1. 25. $\frac{x-a+b}{x+a-b}$.
 16. $\frac{ab}{a^2+4b^2}$. 21. b . 26. 1.
 17. $\frac{(x-2)(x-5)}{x^2}$. 22. $\frac{y}{x-y}$. 27. $\frac{x-y-z}{x+y+z}$.

Exercise 61. Page 126.

1. 1. 2. $\frac{x-4}{x-5}$. 3. $\frac{1}{x+1}$.
 4. $\frac{(x+a)(x-a)}{ax+bx+cx-bc}$. 5. $\frac{(a-x)(a+x)^2(a^2+x^2)}{2a^3x^2}$.
 6. $\frac{xy^2}{(x+y)(x-y)^2}$. 11. $\frac{1+x^2}{1+x}$. 16. x .
 7. $\frac{x^2+1}{2x}$. 12. 1. 17. $\frac{a(a^2+2ab+2b^2)}{(a+b)^2}$.
 8. $\frac{1}{x+1}$. 13. $\frac{x+y}{x^2-2xy+y^2}$. 18. $m-1$.
 9. $\frac{1+x}{1+x^2}$. 14. $\frac{(x+y)^2}{2y}$. 19. $\frac{1}{c(a-b-c)}$.
 10. $x+1$. 15. $\frac{ax}{x+a}$. 20. $\frac{3(4-x)}{7-4x}$.

Exercise 62. Page 127.

1. $\frac{x-8}{x+8}$. 4. $\frac{1}{2(x+1)^2}$. 7. 1. 10. $\frac{2a-b}{a+b}$.
 2. $1\frac{1}{2}$. 5. x . 8. $\frac{xy}{x^2+y^2}$. 11. $\frac{x}{a}$.
 3. 46. 6. 0. 9. 2.

12. $x^2 + 3x + 3 - \frac{3}{x} + \frac{1}{x^2}$ 13. 1. 14. 0. 15. a .
16. $\frac{1}{abc}$ 20. a . 24. $\frac{-bc(c-b)^2}{b^4 + b^2c^2 + c^4}$.
17. $\frac{bc+ac+ab}{bc+ac-ab}$ 21. $a(2a-x-y)$. 25. $\frac{z}{(x-z)(y-z)}$.
18. m . 22. $\frac{4}{3(x+1)}$ 26. $-\frac{1}{c(a-c)(b-c)}$.
19. $\frac{(a+b+c)^2}{2bc}$ 23. $\frac{x^2y}{2}$ 27. $\frac{1}{(x+1)^2}$.

Exercise 63. Page 131.

1. $x=16$. 6. $x=-\frac{1}{8}$. 11. $x=3$. 16. $x=2$.
2. $x=5$. 7. $x=9$. 12. $x=10$. 17. $x=7$.
3. $x=\frac{1}{2}$. 8. $x=1\frac{1}{2}$. 13. $x=6$. 18. $x=9$.
4. $x=1$. 9. $x=7$. 14. $x=12$. 19. $x=9$.
5. $x=8$. 10. $x=-5$. 15. $x=5$.

Exercise 64. Page 133.

1. $x=8$. 3. $x=4$. 5. $x=1\frac{1}{2}$. 7. $x=2$.
2. $x=9$. 4. $x=20$. 6. $x=4$. 8. $x=3$.
9. $x=-2$. 10. $x=11$.

Exercise 65. Page 134.

1. $x=c$. 6. $x=2$. 11. $x=\frac{18a+2b}{3+4a}$.
2. $x=\frac{3c-2a}{5b-c}$ 7. $x=0$. 12. $x=b-1$.
3. $x=\frac{c-d}{a^2-b^2+b-c}$ 8. $x=\frac{b}{a-1}$ 13. $x=0, \frac{b}{c}$.
4. $x=1$. 9. $x=0, \frac{b}{c}$ 14. $x=bm$.
5. $x=-\frac{(a+b)^2}{a-b}$ 10. $x=\frac{3a+1}{2a+b}$ 15. $x=1$.

$$16. x = \frac{2a^3}{b-1}.$$

$$17. x = \frac{a-1}{b}.$$

$$18. x = \frac{bc}{c^2-b}.$$

$$19. x = \frac{ab-1}{bc+d}.$$

$$20. x = \frac{d}{c}.$$

Exercise 66. Page 135.

- | | | | |
|------------------------|------------------------|-----------------------------|--------------------------|
| 1. $x=7$. | 7. $x=8$. | 13. $x=7$. | 19. $x=3\frac{2}{3}$. |
| 2. $x=\frac{1}{3}$ | 8. $x=-1\frac{1}{3}$. | 14. $x=a+\frac{b^2}{a+b}$. | 20. $x=\frac{ab}{a+b}$. |
| 3. $x=-1\frac{1}{2}$. | 9. $x=14$. | 15. $x=c$. | 21. $x=12$. |
| 4. $x=1\frac{1}{2}$. | 10. $x=1\frac{2}{3}$. | 16. $x=0$. | 22. $x=3$. |
| 5. $x=3$. | 11. $x=2$. | 17. $x=1$. | 23. $x=-9$. |
| 6. $x=0$. | 12. $x=-2$. | 18. $x=1$. | 24. $x=1\frac{5}{8}$. |

Exercise 67. Page 137.

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|--|--|---------------------------------------|------------|----------------|
| 1. 24. | 3. 80. | 5. 28, 32. | 7. 76, 24. | 9. 3456, 2304. |
| 2. 168. | 4. 8. | 6. $2\frac{2}{3}$, $47\frac{1}{3}$. | 8. 960. | 10. 36, 9. |
| 11. 420. | 12. 85 gallons wine, 35 gallons of water. | | | |
| 13. 18 pounds saltpeter, 3 pounds sulphur, 3 pounds charcoal. | | | | |
| 14. 28, 18. | 15. House, \$600; garden, \$250. | | | |
| 16. \$7200. | 18. 47, 23. | 21. 19, 20, 21, 22. | | |
| 17. $\frac{a+b}{2}, \frac{a-b}{2}$. | 19. 7, 32. | 22. 42 years. | | |
| | 20. 112, 96. | 23. 20 years. | | |
| 24. Father, 36 years; son, 12 years. | | 27. 2 days. | | |
| 25. A, 28 years; B, 14 years. | | 28. $1\frac{1}{2}$ days. | | |
| 26. Father, 40 years; son, 10 years. | | 29. $26\frac{2}{3}$ days. | | |
| 30. A, 168 days; B, 84 days; C, 42 days. | | 31. $10\frac{1}{2}$ days. | | |
| 32. B and C, $21\frac{1}{11}$ hours; A, B, and C, $10\frac{1}{2}$ hours. | | | | |
| 33. 10 days. | 35. A, $37\frac{1}{2}$ minutes; B, 25 minutes. | | | |
| 34. 40 minutes. | 36. $4\frac{1}{3}$ hours. | 37. 48 minutes. | | |
| 38. $\frac{abc}{ab+ac+bc}$ min. | 39. $48\frac{2}{3}$ minutes. | 41. 7 miles. | | |
| | 40. 42 hours. | | | |

42. $\frac{abc}{b+c}$ miles. 44. 1st, 5 miles per hour; 2d, 3 miles per hour.
43. 160 miles. 45. 175 miles.
46. I. $16\frac{1}{11}$ minutes past three; II. $32\frac{1}{11}$ minutes past six; III. $49\frac{1}{11}$ minutes past nine.
47. I. $32\frac{1}{11}$ minutes past three; II. $5\frac{1}{11}$ and $38\frac{2}{11}$ minutes past four; III. $21\frac{2}{11}$ and $54\frac{1}{11}$ minutes past seven.
48. I. $38\frac{2}{11}$ minutes past one; II. $54\frac{1}{11}$ minutes past four; III. $10\frac{1}{4}$ minutes past eight.
49. $5\frac{1}{11}$ minutes past two. 50. 700 leaps.
51. Greyhound, 450 leaps; hare, 600 leaps.
52. 240 leaps. 53. Length, 13 feet; breadth, 8 feet.
54. $56\frac{1}{2}$ square feet. 55. Length, 27 in.; breadth, 14 in.
56. Length, 15 feet; breadth, 11 feet.
57. 111 pounds tin; 69 pounds of lead. 59. 189.
58. 76 pounds gold; 30 pounds silver. 60. $6\frac{2}{3}$ ounces.
61. 330 feet. 62. \$2284. 63. 24 shots. 64. 60.
65. 12,000 square yards; $56\frac{1}{2}$ cents per square yard.
66. 20 seconds. 67. \$55,500. 68. 103 sheep.
69. \$3700. 70. 10 gal. from 1st cask, 4 gal. from 2d cask.
71. 60. 72. 1504. 73. 39. 74. $9\frac{1}{2}$ miles.
75. $\frac{ab-d}{b+c}$. 76. $\frac{a(b-c)}{b-a}$ of 1st kind; $\frac{b(c-a)}{b-a}$ of 2d kind.

Exercise 68. Page 153.

- | | | | |
|--------------|-------------|-------------|--------------|
| 1. $x=2$. | 4. $x=12$. | 7. $x=29$. | 10. $x=3$. |
| $y=1$. | $y=-3$. | $y=23$. | $y=2$. |
| 2. $x=2$. | 5. $x=6$. | 8. $x=8$. | 11. $x=3$. |
| $y=-1$. | $y=7$. | $y=4$. | $y=20$. |
| 3. $x=2$. | 6. $x=7$. | 9. $x=7$. | 12. $x=-3$. |
| $y=8$. | $y=9$. | $y=14$. | $y=-2$. |
| 13. $x=-2$. | 14. $x=7$. | | |
| $y=1$. | $y=-2$. | | |

Exercise 69. Page 155.

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|-------------|------------|----------------------|------------|
| 1. $x=10.$ | 4. $x=4.$ | 7. $x=20.$ | 10. $x=2.$ |
| $y=7.$ | $y=1.$ | $y=10.$ | $y=2.$ |
| 2. $x=17.$ | 5. $x=5.$ | 8. $x=2.$ | 11. $x=2.$ |
| $y=19.$ | $y=5.$ | $y=-3.$ | $y=3.$ |
| 3. $x=2.$ | 6. $x=21.$ | 9. $x=8.$ | 12. $x=9.$ |
| $y=13.$ | $y=12.$ | $y=4.$ | $y=-3.$ |
| 13. $x=-5.$ | | 14. $x=\frac{1}{2}.$ | |
| $y=14.$ | | $y=-\frac{1}{8}.$ | |

Exercise 70. Page 156.

- | | | | |
|------------|-----------|-------------|-------------|
| 1. $x=8.$ | 4. $x=2.$ | 7. $x=12.$ | 10. $x=4.$ |
| $y=3.$ | $y=13.$ | $y=3.$ | $y=3.$ |
| 2. $x=6.$ | 5. $x=8.$ | 8. $x=2.$ | 11. $x=-2.$ |
| $y=3.$ | $y=1.$ | $y=6.$ | $y=19.$ |
| 3. $x=6.$ | 6. $x=3.$ | 9. $x=3.$ | 12. $x=7.$ |
| $y=8.$ | $y=4.$ | $y=5.$ | $y=-5.$ |
| 13. $x=2.$ | | 14. $x=11.$ | |
| $y=-3.$ | | $y=10.$ | |

Exercise 71. Page 157.

- | | | | |
|------------|------------------------|-----------------------|------------------------|
| 1. $x=1.$ | 7. $x=11.$ | 13. $x=12.$ | 19. $x=4.$ |
| $y=7.$ | $y=6.$ | $y=6.$ | $y=-1.$ |
| 2. $x=2.$ | 8. $x=3.$ | 14. $x=3.$ | 20. $x=2.$ |
| $y=3.$ | $y=2.$ | $y=2.$ | $y=3.$ |
| 3. $x=1.$ | 9. $x=7.$ | 15. $x=9.$ | 21. $x=6.$ |
| $y=8.$ | $y=5.$ | $y=8.$ | $y=5.$ |
| 4. $x=9.$ | 10. $x=-5\frac{1}{2}.$ | 16. $x=5.$ | 22. $x=19\frac{1}{2}.$ |
| $y=8.$ | $y=-9\frac{1}{2}.$ | $y=7.$ | $y=-17.$ |
| 5. $x=-3.$ | 11. $x=7.$ | 17. $x=7.$ | 23. $x=4.$ |
| $y=-2.$ | $y=8.$ | $y=2.$ | $y=3.$ |
| 6. $x=7.$ | 12. $x=14.$ | 18. $x=9\frac{1}{2}.$ | 24. $x=7.$ |
| $y=10.$ | $y=46.$ | $y=2.$ | $y=9.$ |
| | 25. $x=21.$ | | |
| | $y=20.$ | | |

Exercise 72. Page 160.

$$1. x = \frac{a+b}{2}.$$

$$y = \frac{a-b}{2}.$$

$$2. x = \frac{cq-br}{aq-bp}.$$

$$y = \frac{ar-cp}{aq-bp}.$$

$$3. x = \frac{bn-aq}{np-mq}.$$

$$y = \frac{ap-bm}{np-mq}.$$

$$4. x = \frac{ce-bd}{a(c-b)}.$$

$$y = \frac{e-d}{b-c}.$$

$$5. x = \frac{nr' + n'r}{mn' + m'n}.$$

$$y = \frac{mr' - m'r}{mn' + m'n}.$$

$$6. x = \frac{c(f-bc)}{af-bd}.$$

$$y = \frac{c(ac-d)}{af-bd}.$$

$$7. x = \frac{ab^2c^2}{b^2c-a^2}.$$

$$y = \frac{a^2bc}{a^2-b^2c}.$$

$$8. x = \frac{1}{ab}.$$

$$y = \frac{1}{cd}.$$

$$9. x = \frac{2b^2-6a^2+d}{3a}.$$

$$y = \frac{3a^2-b^2+d}{3b}.$$

$$10. x = \frac{a}{a-b}.$$

$$y = \frac{b}{a+b}.$$

$$11. x = a-b.$$

$$y = a+b.$$

$$12. x = \frac{ab-1}{(a-1)(b-1)}.$$

$$y = \frac{a-b}{(a-1)(b-1)}.$$

$$13. x = \frac{a+b}{2}.$$

$$y = \frac{a-b}{2}.$$

$$14. x = \frac{c^2-a^2+b^2}{2a}.$$

$$y = \frac{c^2+a^2-b^2}{2b}.$$

$$15. x = (a+b)^2.$$

$$y = (a-b)^2.$$

$$16. x = a+c.$$

$$y = b+c.$$

$$17. x = a - \frac{abc^2}{ac+b}.$$

$$y = \frac{b^2}{ac+b}.$$

$$18. x = a + b.$$

$$y = a - b.$$

$$19. x = b - a.$$

$$y = a - b.$$

$$20. x = \frac{ab(a+b)}{2(a^2+b^2)}.$$

$$y = \frac{a^3 - b^3}{2(a^2 + b^2)}.$$

Exercise 73. Page 162.

$$1. x = \frac{1}{2}.$$

$$y = \frac{1}{4}.$$

$$4. x = \frac{1}{2}.$$

$$y = 1.$$

$$7. x = \frac{1}{a}.$$

$$y = \frac{1}{b}.$$

$$2. x = \frac{1}{b-2a}.$$

$$5. x = -1.$$

$$y = -\frac{1}{2}.$$

$$y = \frac{2}{3a-b}.$$

$$6. x = \frac{ab(a+b)}{c(a^2+ab+b^2)}.$$

$$8. x = \frac{1}{n}.$$

$$3. x = 6.$$

$$y = 9.$$

$$y = -\frac{a+b}{c}.$$

$$y = \frac{1}{m}.$$

$$9. x = \frac{a^2 + b^2}{am + bn}.$$

$$y = \frac{a^2 + b^2}{bm - an}.$$

Exercise 74. Page 164.

$$1. x = 2.$$

$$y = 2.$$

$$z = 2.$$

$$4. x = 5.$$

$$y = 6.$$

$$z = 7.$$

$$7. x = 20.$$

$$y = 10.$$

$$z = 5.$$

$$10. x = 4.$$

$$y = 0.$$

$$z = 5.$$

$$2. x = 5.$$

$$y = 6.$$

$$z = 8.$$

$$5. x = 1.$$

$$y = 4.$$

$$z = 6.$$

$$8. x = \frac{2}{3}.$$

$$y = -7.$$

$$z = 36\frac{1}{3}.$$

$$11. x = 5.$$

$$y = -5.$$

$$z = 5.$$

$$3. x = 1.$$

$$y = 2.$$

$$z = 3.$$

$$6. x = 2.$$

$$y = 9.$$

$$z = 10.$$

$$9. x = 2.$$

$$y = 3.$$

$$z = 1.$$

$$12. x = 45.$$

$$y = -21.$$

$$z = 1.$$

13. $x=1.$ 15. $x=-1\frac{1}{2}.$ 17. $x=\frac{1}{8}.$ 19. $x=6.$
 $y=2.$ $y=2\frac{1}{2}.$ $y=\frac{1}{4}.$ $y=-12.$
 $z=-3.$ $z=6\frac{1}{2}.$ $z=\frac{1}{8}.$ $z=18.$
14. $x=1.$ 16. $x=-4.$ 18. $x=9.$ 20. $x=1.$
 $y=-1.$ $y=-3\frac{2}{3}.$ $y=11.$ $y=\frac{1}{2}.$
 $z=2.$ $z=-4\frac{1}{2}.$ $z=13.$ $z=\frac{1}{8}.$
21. $x=\frac{2}{a+b}.$ 22. $x=\frac{b^2+c^2-a^2}{2bc}.$
 $y=\frac{2}{a+c}.$ $y=\frac{a^2-b^2+c^2}{2ac}.$
 $z=\frac{2}{b+c}.$ $z=\frac{a^2+b^2-c^2}{2ab}.$
23. $x=\frac{1}{2}.$ 24. $x=3\frac{1}{8}.$ 25. $x=1\frac{1}{8}.$
 $y=\frac{1}{8}.$ $y=1\frac{3}{4}.$ $y=-3\frac{1}{2}.$
 $z=\frac{1}{4}.$ $z=1\frac{9}{16}.$ $z=2\frac{1}{16}.$
26. $x=\frac{a+b}{2a}.$
 $y=\frac{ab+ac+bc-b^2-2c^2}{2(b^2-c^2)}.$ 27. $x=17.$
 $z=\frac{3bc-ab-ac-b^2}{2(b^2-c^2)}.$ $y=22.$
 $z=-25.$
28. $x=-(a+b).$ $y=\frac{(a-b-c)(a+b)}{a+b+c}.$ $z=\frac{(a+b-c)(a+b)}{a+b+c}.$

Exercise 75. Page 167.

1. 41, 7. 2. 35, 11, 98. 3. 6, 4. 4. 4.78, 1.0095.
5. Father, 35 years; son, 14 years.
6. Father, 40 years; son, 10 years.
7. A, \$116; B, \$166.
8. Wheat, \$1 $\frac{1}{4}$ per bushel; barley, \$ $\frac{3}{4}$ per bushel.
9. A, \$31; B, \$27. 10. Horse, \$100; cow, \$50.
11. $\frac{5}{8}.$ 14. $\frac{3}{15}.$ 17. 28. 20. 75. 23. 717.
12. $\frac{9}{11}.$ 15. $\frac{2}{3}, \frac{5}{8}.$ 18. 24. 21. 36. 24. 759.
13. $\frac{2}{11}.$ 16. $\frac{2}{3}.$ 19. 84. 22. 69. 25. 126.

26. 432.
27. $7\frac{1}{2}$ hours, $4\frac{1}{2}$ hours.
28. 3 miles per hour.
29. Distance, 20 miles; rate per hour, 8 miles.
30. Time rowing down, 4 hours; time rowing up, 6 hours; rate of stream per hour, $\frac{1}{4}$ mile.
31. 20 pounds @ 42 cents; 10 pounds @ 54 cents.
32. 40 pounds @ 90 cents; 90 pounds @ 28 cents.
33. Rye, 20 bushels; wheat, 52 bushels.
34. A, $\$3\frac{3}{4}$; B, $\$3$; C, $\$2\frac{1}{2}$.
35. A, $1\frac{1}{4}$ hours; B, $3\frac{1}{2}$ hours; C, 7 hours.
36. 54 boxes; 36 bales.
37. 1st, 36 days; 2d, 45 days.
38. 21 crowns; 63 guineas.
39. A, 20 days; B, 30 days; C, 60 days.
40. A, $\frac{2abc}{ac+bc-ab}$ minutes;
41. $\$5000$ @ 5%.
42. $\$5000$ @ 4%.
- B, $\frac{2abc}{ab+bc-ac}$ minutes;
43. $\$20,000$ @ 5%.
- C, $\frac{2abc}{ab+ac-bc}$ minutes.
44. $\$25,000$ @ 6%.
44. Sum, $\frac{an-bm}{n-m}$ dollars; rate, $\frac{100(b-a)}{an-bm}$.
45. Sum, $\frac{ad-bc}{a-b}$ dollars; rate, $\frac{1200(c-d)}{ad-bc}$.
46. 1st, $\$3000$ @ 4%; 2d, $\$4000$ @ 5%; 3d, $\$4500$ @ 6%.
47. $\$6750$ @ 3%; $\$6000$ @ 5%.
48. 3 per cents at $\$60$; 4 per cents at $\$75$.
49. 3 per cents at $\$75$; $3\frac{1}{2}$ per cents at $\$88$.
50. $\$2000$ @ 2%; $\$500$ @ 4%.
51. 2660 square yards.
52. Length, 14 feet; breadth, 10 feet.
53. Length, 90 yards; breadth, 60 yards.
54. A, 9 miles; B, $8\frac{1}{11}$ miles.
55. A, 12 miles; B, 10 miles.

56. Express-train, 45 miles; accommodation-train, 30 miles. 57. 315 miles. 58. 30 miles.
 59. $33\frac{1}{4}$ miles per hour.
 60. Ox, \$105; sheep, $\$8\frac{3}{4}$; lamb, $\$5\frac{1}{4}$.
 61. 2 horses, 4 oxen, 94 sheep.
 62. A, \$50; B, \$30; C, \$20.
 63. A, $\$9\frac{3}{4}$; B, $\$5\frac{1}{4}$; C, \$3.
 64. A, $\$3\frac{1}{2}$; B, $\$2\frac{1}{2}$; C, $\$1\frac{1}{2}$.

Exercise 76. Page 183.

1. a^6 . 2. x^{15} . 3. x^4y^6 . 4. $\frac{a^{12}b^8}{16}$. 5. $\frac{243x^{10}y^5}{32a^{15}b^{10}}$.
6. $x^3 + 6x^2 + 12x + 8$. 7. $x^4 - 8x^3 + 24x^2 - 32x + 16$.
 8. $x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$.
 9. $1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$.
10. $8m^3 - 12m^2 + 6m - 1$. 12. $-125a^3x^9y^6$.
11. $16a^8b^4c^{12}$. 13. $49m^6n^2x^4y^8$.
14. $-\frac{32x^{15}y^5}{243a^5b^5c^5}$. 15. $81x^4 + 108x^3 + 54x^2 + 12x + 1$.
 16. $16x^4 - 32ax^3 + 24a^2x^2 - 8a^3x + a^4$.
17. $243x^5 + 810ax^4 + 1080a^2x^3 + 720a^3x^2 + 240a^4x + 32a^5$.
18. $16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$.
19. $x^{12}y^6 - 12x^{11}y^7 + 60x^{10}y^8 - 160x^9y^9 + 240x^8y^{10}$
 $- 192x^7y^{11} + 64x^6y^{12}$.
20. $a^7b^7 - 21a^6b^6 + 189a^5b^5 - 945a^4b^4 + 2835a^3b^3$
 $- 5103a^2b^2 + 5103ab - 2187$.
21. $-243a^{10}b^{10}c^5$. 22. $729x^6y^{18}$. 23. $-3125a^{10}b^5x^{15}$.
24. $\frac{81a^4b^8}{256c^{12}}$. 25. $-\frac{x^{14}y^{21}z^{28}}{128}$. 26. $1 - 2a - a^2 + 2a^3 + a^4$.
27. $8 - 36x + 102x^2 - 171x^3 + 204x^4 - 144x^5 + 64x^6$.
28. $1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6$.
29. $1 - 3x + 6x^2 - 7x^3 + 6x^4 - 3x^5 + x^6$.
30. $1 + 4x + 10x^2 + 16x^3 + 19x^4 + 16x^5 + 10x^6 + 4x^7 + x^8$.

Exercise 77. Page 185.

- $$\begin{array}{llll}
 1. \pm a^2, \pm x^2, \pm 2a^3b, 4, ax^3y^3, \pm 2a^3bc^2, -2a^3. & & & \\
 2. -12c^2d^4xy^3, 15b^7z^5, \pm 42c^4z. & 4. 2ab^2c. & 7. 170. & \\
 3. \pm 231b^2c^4y^6z^8, -\frac{6bc^5}{7z^8}, \pm \frac{2x^3}{3z^5}. & 5. 36x^3y^3z^2. & 8. 16. & \\
 & 6. 92. & 9. 1. &
 \end{array}$$

Exercise 78. Page 187.

1. $a^2 + 2a - 1.$
2. $x^2 - xy + y^2.$
3. $2a^3 - 3a^2x - ax^2.$
4. $3x^3 - 4xy^2 - 2y^3.$
5. $2a^4 + 4a^2c^2 - 4c^4.$
6. $2x^2 - 5x + 3.$
7. $4x^2 - 2ab + 2b^2.$
8. $4 - 3x + 2x^2 - x^3.$
9. $x^3 - 2x^2y + 2xy^2 - y^3.$
10. $2 - 3a - a^2 + 2a^3.$
11. $3a - b + 5c + d.$
12. $5x^3 - 3x^2y - 4xy^2 + y^3.$
13. $m^4 - 2m^3 + 3m^2 - 4m + 5.$
14. $x^3 - \frac{1}{2}xy - y^2.$
15. $x^2 - 2xy + y^2 - \frac{y^3}{x}.$
16. $\frac{a^2}{3} - \frac{2}{3}ax + \frac{x^2}{2}.$
17. $1 + \frac{2}{x} + \frac{3}{x^2} + \frac{4}{x^3}.$
18. $\frac{a}{b} - 1 + \frac{b}{a}.$
19. $x^3 + \frac{x}{2} - \frac{1}{x}.$

Exercise 79. Page 190.

1. 347; 69.4; 43.21; 37.89; 8.008.
2. 129.63; 2.1319; 0.4937; 0.75416.
3. 0.9486.....; 2.4919.....; 0.6557.....; 0.0923.....;
4.1231.....; 11.3578.....; 18.6348.....
4. 119.5633.....; 1.5811.....; 44.7213.....; 0.5477.....;
0.1732.....; 10.5356.....
5. 0.0333.....; 0.0632.....; 0.0707.....; 1.4142.....;
2.2360.....; 1.8027.....; 2.9325.....
6. $\frac{1}{2}$; $\frac{4}{7}$; $\frac{5}{8}$; $\frac{1}{3}$; $\frac{1}{7}$; $\frac{4}{7}$.
7. 0.7071.....; 0.8164.....; 0.8660.....; 0.1767.....;
0.2338.....; 0.2190.....; 0.9258.....; 0.2886.....

Exercise 80. Page 192.

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|-------------------|------------------|-----------------------|
| 1. $x+2y$. | 5. x^2+x+1 . | 9. $1-x+x^2-x^2$. |
| 2. $a-3$. | 6. $1-3x+4x^2$. | 10. $a^2+3ab-9b^2$. |
| 3. $x+4$. | 7. a^2-2a-1 . | 11. $c^2-4bc+4b^2$. |
| 4. x^2-ax-a^2 . | 8. $4x^2+4x-1$. | 12. $2a^2+4ab-3b^2$. |

Exercise 81. Page 195.

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|--|-------------|------------|-------------|------------|
| 1. 65. | 4. 9.6. | 7. 11.7. | 10. 192. | 13. 32.1. |
| 2. 48. | 5. 478. | 8. 108. | 11. 2.34. | 14. 46.8. |
| 3. 64. | 6. 114. | 9. 1.41. | 12. 3.84. | 15. 9.36. |
| 16. 20.53. | | 17. 1.111. | | 18. 27.55. |
| | 19. 4.5045. | | 20. 1.7479. | |
| 21. 1.3572.....; 0.5848.....; 0.2154.....; 1.5874.....;
0.7368..... | | | | |

Exercise 82. Page 195.

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|--------------|----------------|------------|-------------|
| 1. $3a-5b$. | 2. $1-x+x^2$. | 3. $2-x$. | 4. $3x-1$. |
| | 5. $1-y$. | | |

Exercise 83. Page 197.

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|-------------------------------------|------------------------|----------------------------------|----------------------------------|
| 1. ± 7 . | 4. $\pm \frac{1}{2}$. | 7. ± 1 . | 10. $\pm 2\frac{1}{2}$. |
| 2. ± 3 . | 5. $\pm \frac{1}{2}$. | 8. ± 5 . | 11. ± 5 . |
| 3. $\pm \sqrt{3}$. | 6. $\pm \sqrt{11}$. | 9. ± 3 . | 12. ± 2 . |
| 13. ± 2 . | | 15. $\pm \sqrt{\frac{q-n}{m}}$. | 16. $\pm \sqrt{\frac{b}{a-1}}$. |
| 14. $\pm \sqrt{-\frac{a}{1+b^2}}$. | | | |

Exercise 84. Page 201.

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|--------------------------------------|-------------------------------------|---------------------------------------|---------------------------|
| 1. 2, -6. | 6. $\frac{1}{4}$, $-\frac{1}{8}$. | 11. 6, $-1\frac{1}{8}$. | 16. 11, 0. |
| 2. 8, -2. | 7. 3, -2. | 12. 2, $\frac{8}{9}$. | 17. 3, -1. |
| 3. $11\frac{1}{2}$, $\frac{1}{2}$. | 8. 1, $-\frac{2}{3}$. | 13. $\pm \sqrt{10}$. | 18. $1 \pm \sqrt{3}$. |
| 4. 8, -1. | 9. 14, $-\frac{1}{2}$. | 14. $\frac{1}{2}$, $-4\frac{3}{8}$. | 19. -3, $-3\frac{3}{8}$. |
| 5. $2\frac{1}{8}$, -1. | 10. $\frac{1}{2}$, $\frac{1}{8}$. | 15. 3, 0. | 20. 5, $1\frac{1}{4}$. |

21. 10, -29. 23. 4, -1. 25. $2\frac{2}{3}$, 0. 27. 5, 2.
 22. 3, $-\frac{5}{7}$. 24. 3, $-\frac{4}{3}$. 26. 7, -2. 28. 3, $\frac{3}{4}$.
 29. 5, $-4\frac{1}{2}$. 30. 13, $\pm \sqrt{3_{11}^2}$.

Exercise 85. Page 204.

1. $-a \pm a\sqrt{2}$. 3. $\frac{m}{2}$, $-\frac{7m}{2}$. 4. $3n$, $-\frac{n}{2}$.
 2. $2a \pm a\sqrt{11}$.
 5. $\frac{a(a+b)}{a-b}$, $\frac{a(a-b)}{a+b}$. 6. $\frac{c \pm \sqrt{c^2+4ac}}{2(a+b)}$. 7. $\frac{b^2}{ac}$.
 8. a , $\frac{1}{a}$. 9. $\frac{2ab-ac-bc}{a+b-2c}$, 0. 10. $-a$, $-\frac{b}{a}$.
 11. $1 \pm \sqrt{-2}$.
 12. $a^2+b^2 \pm (a-b)\sqrt{a^2+b^2}$. 14. 1, $-a$.
 13. $\frac{a}{2}+b$, $\frac{a-b}{2}$. 15. b , $-\frac{a}{b}$.
 16. $\pm \sqrt{ab}$.
 17. a , $-\frac{b(a+b)}{2a+b}$. 18. $4a$, $-\frac{a}{4}$. 19. $\frac{6 \pm 3\sqrt{a^2+4}}{a}$.
 20. $\frac{m}{n}$, $-\frac{n}{m^2}$. 21. $\frac{a^2+b^2 \pm \sqrt{(a^2-b^2)^2+4abc^2}}{2ab}$. 22. ± 1 .
 23. $\pm m\sqrt{\frac{c-2}{c}}$. 31. $2a$, $\frac{a+b}{2}$.
 24. $\frac{1-3a \pm 2a\sqrt{a}}{(a-1)^2}$. 32. $a-b$, c .
 33. $m \pm (n-p)$.
 25. $\frac{a+b}{a-b}$, $\frac{a-b}{a+b}$. 34. $\frac{m+n \pm (p+q)}{2}$.
 26. $(m+n)^2$, $-(m-n)^2$. 35. $m+n$, $\frac{m+n}{mn}$.
 27. $\frac{4a-5b}{6ab}$, $\frac{a-2b}{3ab}$. 36. $2a-b$, $a+2b$.
 28. $\frac{2a-b}{ac}$, $-\frac{3a+2b}{bc}$. 37. $\frac{a+b}{a-b}$, $\frac{a-b}{a+b}$.
 29. $a+\frac{m}{2}$, $m-2a$. 38. $2a+6b$, $a-8b$.
 30. $\frac{3a-2b}{6}$, $\frac{2a-3b}{6}$. 39. $\frac{a}{2}-3b$, $-(5a+b)$.

40. $\frac{4a^2+9b^2}{2a+3b}, -(2a+3b).$ 43. $1, \frac{b(c-d)}{c(a-b)}.$
 41. $1, \frac{m-n}{n-p}.$ 44. $-1, \frac{a-2b}{2a-3b}.$
 42. $1, \frac{a}{a+b+c}.$ 45. $\frac{a-b}{a-2b}, -\frac{a+b}{a+2b}.$

Exercise 86. Page 208.

1. $\pm 1, \pm 2.$ 3. $0, -1 \pm \sqrt{6}.$
 2. $1, 2, -3, 4.$ 4. $0, 5, -7.$
 5. $-2, 3, -4, 5.$
 6. $a, \frac{1}{2}(-a-3 \pm \sqrt{-3a^2-6a+1}).$
 7. $1, -1, 1.$ 10. $\pm 1, \frac{1}{2}(\pm 1 \pm \sqrt{-3}).$
 8. $\frac{1}{2}, \frac{1}{4}(-1 \pm \sqrt{-3}).$ 11. $0, a, \pm b.$
 9. $-\frac{1}{2}, \frac{1}{4}(1 \pm \sqrt{-3}).$ 12. $-1, \frac{1}{2} \pm \frac{1}{2n} \sqrt{-3n^2-4n}.$

Exercise 87. Page 211.

1. $x^2-3x+2=0.$ 5. $2x^2+11x+5=0.$
 2. $x^2-4x-21=0.$ 6. $63x^2-32x-63=0.$
 3. $6x^2-5x+1=0.$ 7. $16x^4-153x^2+81=0.$
 4. $6x^2+5x-6=0.$ 8. $x(x^3-6x^2+11x-6)=0.$
 9. $x(4x^3-7x-3)=0.$
 10. $x^2-4ax+3a^2-4ab-4b^2=0.$
 11. $x^2+ax-6a^2+5ab-b^2=0.$
 12. $x^2-a^2x-x-a^3+a=0.$
 13. Rational and both positive. 16. Imaginary.
 14. Rational and of opposite signs. 17. Surds and both
 15. Rational and of opposite signs. negative.
 18. Imaginary. 19. Rational and of opposite signs.
 20. Rational, equal in value, and both negative.

Exercise 88. Page 212.

1. Max., 13.
2. Min., $4a$.
3. Max., $+\infty$; min., $-\infty$.
4. Max., $\frac{(a-b)^2}{4}$.
5. Max., 1.
6. Min., 4.
7. Min., 8.
8. Min., $\frac{4ab}{(a+b)^2}$.
9. Max., $+\frac{1}{2}\sqrt{\frac{1}{a}}$.
10. Each 10 inches.
11. Each 10 inches.
12. $\frac{1}{2}$.

Exercise 89. Page 214.

1. $1, -2, 1 \pm \sqrt{-3}, \frac{1}{2}(-1 \pm \sqrt{-3})$.
2. $\pm 1, \pm 2$.
3. $\pm 3, \pm \frac{1}{2}$.
4. $\pm 1, \pm \sqrt{-1}, \pm \frac{1}{2}, \pm \sqrt{-\frac{1}{2}}$.
5. $1, \frac{1}{2}$.
6. $\pm 2, \pm \frac{1}{2}$.
7. $\pm \sqrt[2n]{2\frac{1}{2}}, \pm \sqrt[2n]{-\frac{5}{8}}$.
8. $\pm 5, \pm 2$.
9. $\sqrt[3]{-2}, \sqrt[3]{-12}$.
10. $0, \frac{1}{8}, -\frac{1}{2}, \frac{1}{8}(-1 \pm \sqrt{-3}), \frac{1}{4}(1 \pm \sqrt{-3})$.
11. $\pm \sqrt[4]{-1}, \pm \sqrt[4]{-21}$.
12. $1, \sqrt[m]{-4}$.
13. $3, 2, \pm \frac{1}{2}$.
14. $\sqrt[4]{\frac{3}{40}} \pm \sqrt[4]{\frac{1}{40}} \sqrt{89}$.
15. $1, 1, -3, 5$.
16. $-1, 2, -3, 4$.
17. $1\frac{1}{2}, \pm \frac{1}{8}, \frac{1}{2}$.
18. $\pm 1, \pm 3$.

Exercise 90. Page 216.

1. 10, 11, 12.
2. 3, 4.
3. 1, 2, 3.
4. 12.
5. 6.
6. 12.
7. \$11\frac{1}{2}.
8. 31\frac{1}{2} cents.
9. 12 inches.
10. 121 yards, 120 yards.
11. 16, 16, 28.
12. $1\frac{1}{2}$ yards.
13. 2.4142..... inches.
14. 7.63....., 12.36.....
15. A, 36 hours; B, 45 hours.
16. 5 hours, 7 hours.
17. 3 hours, 5 hours.
18. 8 feet; $4\frac{1}{2}$ pounds.

19. 39. 21. 5 miles per hour. 23. \$80.
 20. 15, 20. 22. 4550. 24. \$60,
 25. 100 shares @ \$75. or \$40.
 26. 15, 9, and 6 yards; or, $2\frac{2}{3}$, $1\frac{1}{3}$, and $\frac{2}{3}$ yards.
 27. $5\frac{5}{8}$ miles per hour.

Exercise 91. Page 221.

- | | | | |
|------------|------------------------------------|------------|--------------------------------|
| 1. 9, 4. | 6. 20, -6. | 11. 40, 9. | 16. 4, -2 $\frac{3}{4}$. |
| 4, 9. | 6, -20. | 9, 40. | 1, -1 $\frac{1}{4}$. |
| 2. 25, 4. | 7. 10, 2. | 12. 6, 5. | 17. 3, -3 $\frac{1}{4}$. |
| 4, 25. | 2, 10. | 5, 6. | 4, -3 $\frac{1}{8}$. |
| 3. 22, -3. | 8. 4, 2. | 13. 10, 2. | 18. $\frac{1}{8}$. |
| 3, -22. | 2, 4. | 2, 10. | $\frac{1}{2}$. |
| 4. 50, -5. | 9. $\frac{1}{8}$, $\frac{1}{2}$. | 14. 5, -3. | 19. $\frac{1}{8}$. |
| 5, -50. | $\frac{1}{2}$, $\frac{1}{8}$. | 3, -5. | 2. |
| 5. 13, -3. | 10. 3, -1 $\frac{3}{4}$. | 15. 7, -4. | 20. $\pm\sqrt{2\frac{1}{2}}$. |
| 3, -13. | -4, $8\frac{1}{2}$. | 4, -7. | $2 \mp \sqrt{2\frac{1}{2}}$. |

Exercise 92. Page 222.

- | | | |
|--|---------------------------------------|--------------------------------|
| 1. $\pm 3, \pm 8$. | 4. $\pm 5\sqrt{\frac{3}{7}}$. | 8. $\pm 2, \pm 2\frac{1}{2}$. |
| ± 5 . | $\pm \sqrt{\frac{3}{7}}$. | $\pm 3, \pm 2\frac{2}{3}$. |
| 2. $\pm 2, \pm \sqrt{\frac{3}{2}}$. | 5. $\pm 7, \pm 5\sqrt{\frac{1}{2}}$. | 9. ± 3 . |
| $\pm \frac{1}{2}, \pm 2\sqrt{\frac{3}{2}}$. | $\pm 2, \mp 9\sqrt{\frac{1}{2}}$. | ± 4 . |
| 3. $\pm 4, \pm 3\sqrt{3}$. | 6. $\pm 2, \pm \sqrt{2}$. | 10. $\pm 3, \pm 11\sqrt{-1}$. |
| $\pm 5, \pm \sqrt{3}$. | $\pm 4, \pm 3\sqrt{2}$. | $\pm 1, \mp 18\sqrt{-1}$. |
| | 7. $\pm 3, \pm 5\sqrt{\frac{1}{2}}$. | |
| | $\pm 2, \pm \sqrt{\frac{1}{2}}$. | |

Exercise 93. Page 224.

- | | | |
|------------------------------|--|---------------------|
| 1. 4, 2, $3 \pm \sqrt{21}$. | 3. 4, -2, 0. | 5. $\pm 3, \pm 2$. |
| 2, 4, $3 \mp \sqrt{21}$. | 2, -4, 0. | $\pm 2, \pm 3$. |
| 2. 3, 2, $-3 \pm \sqrt{3}$. | 4. 4, 2, $\frac{1}{8}(-13 \pm \sqrt{377})$. | 6. 3, 2. |
| 2, 3, $-3 \mp \sqrt{3}$. | 2, 4, $\frac{1}{8}(-13 \mp \sqrt{377})$. | 2, 3. |

Exercise 94. Page 224.

1. 4, 3.
-3, -4.
2. $\pm 5, \pm 7\sqrt{\frac{1}{2}}$.
 $\pm 2, \pm 3\sqrt{\frac{1}{2}}$.
3. 8, -3.
 $1\frac{1}{2}, -4$.
4. $\pm 7, \pm 1$.
 $\pm 1, \pm 7$.
5. 2, $-2\frac{2}{3}$.
-1, $-2\frac{1}{3}$.
6. 8, 1.
-1, -8.
7. $\pm \frac{1}{3}$.
 $\pm \frac{1}{4}$.
8. $2\frac{1}{2}, -1\frac{1}{2}$.
 $1\frac{1}{2}, -2\frac{1}{2}$.
9. $\pm 1, \pm \sqrt{-\frac{3}{8}}$.
 $\pm \frac{1}{2}, \mp \frac{3}{2}\sqrt{-\frac{3}{8}}$.
10. 4.
-4.
11. $\frac{1}{4}(\pm 3 \pm \sqrt{29})$.
 $\frac{1}{4}(\pm 3 \mp \sqrt{29})$.
12. $\pm \frac{2}{3}, \pm \frac{7}{3}\sqrt{\frac{1}{18}}$.
 $\mp \frac{1}{3}, \pm \frac{10}{3}\sqrt{\frac{1}{18}}$.
34. $\frac{a}{2}$.
 $\frac{b}{2}$.
13. $\frac{a}{2} \pm b$.
 $\frac{a}{2} \mp b$.
14. 3, -2.
2, -3.
15. $\pm 4, \pm 10\frac{5}{8}$.
 $\pm 9, \pm 2\frac{3}{8}$.
16. 4, 2.
2, 4.
17. 10, $-10\frac{7}{8}$.
15, $-16\frac{1}{8}$.
18. 3, -2.
2, -3.
19. 12, 10.
10, 12.
20. $\frac{1}{2}(a \pm \sqrt{2b^2 - a^2})$.
 $\frac{1}{2}(a \mp \sqrt{2b^2 - a^2})$.
21. $\pm 1\frac{1}{2}, \pm \frac{1}{2}\sqrt{3}$.
 $\pm 1\frac{1}{2}, \pm \frac{1}{2}\sqrt{3}$.
22. ± 6 .
 ± 3 .
23. $\frac{1}{2}, \frac{1}{8}$.
 $\frac{1}{8}, \frac{1}{2}$.
35. 3, 2, $\frac{1}{4}(-3 \pm \sqrt{34\frac{1}{8}})$.
2, 3, $\frac{1}{4}(-3 \mp \sqrt{34\frac{1}{8}})$.
37. 6, $-3\frac{1}{2}$.
4, $-3\frac{7}{10}$.
24. $\pm 3, \pm 2$.
 $\pm 2, \pm 3$.
25. 3, 1, $2 \pm 5\sqrt{-1}$.
1, 3, $2 \mp 5\sqrt{-1}$.
26. a, 0.
0, -a.
27. $\pm \frac{a^2 + b^2}{a - b}$.
 $\pm \frac{2ab}{a - b}$.
28. $\pm (a + b)$.
 $\pm (a - b)$.
29. $\pm 4, 0$.
0, ± 4 .
30. $\pm 4, \pm 3$.
 $\pm 3, \pm 4$.
31. $a + b, 0$.
 $a + b, 0$.
32. $5, \frac{3}{4}$.
3, $-1\frac{1}{4}$.
33. $6\frac{1}{2}, -2\frac{7}{8}$.
 $1\frac{1}{2}, -\frac{9}{8}$.
36. 5, $-2, \frac{1}{2}(3 \pm \sqrt{-67})$.
2, $-5, \frac{1}{2}(-3 \mp \sqrt{-67})$.
38. $\pm \left(\frac{3b - 2a}{5}\right), \pm a\sqrt{-\frac{1}{5}}$.
 $\pm \left(\frac{3a - 2b}{5}\right), \mp b\sqrt{-\frac{1}{5}}$.

Exercise 95. Page 226.

1. 7, 5.
2. 34, 43.
3. $\frac{1}{2}(3 \pm \sqrt{5})$, $\frac{1}{2}(1 \pm \sqrt{5})$.
4. ± 16 , ± 10 .
5. $\frac{2}{3}$, $\frac{2}{3}$.
6. 15 miles; 3, $2\frac{1}{2}$, and 4 miles.
7. 73, 37.
8. 16, 12.
9. 88 yards, 55 yards.
10. 290 yards.
11. 8 feet, 10 feet.
12. \$3000, 7%; \$3500, 6%.

Exercise 96. Page 231.

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|---------------|---|---------------------|
| 1. 19, 8. | 12. 49. | 23. 6, 12, 18, 24. |
| 1, 3. | 22. | 56, 42, 28, 14. |
| 2. 1, 4. | 13. 32. | 8, 16, 24, 32. |
| 11, 4. | 21. | 24. 10. |
| 3. 5. | 14. 19. | 14. |
| 4. | 476. | 176. |
| 4. 9, 19. | 15. $\frac{1}{12}$, $\frac{3}{12}$, $\frac{5}{12}$, $\frac{7}{12}$. | 25. 149. |
| 2, 1. | $\frac{11}{18}$, $\frac{8}{18}$, $\frac{5}{18}$, $\frac{2}{18}$. | 26. 6, 13. |
| 5. 15, 7. | 16. 8. | 17, 7. |
| 2, 5. | 17. 59. | 27. 14 half-crowns. |
| 6. 4, 9. | 18. 3, 6. | 9 4-penny pieces. |
| 13, 5. | 21, 2. | 29. 19 oxen. |
| 7. 6. | 16, 32. | 1 sheep. |
| 2. | 19. 301. | 80 hens. |
| 8. 28, 18, 8. | 20. 91, 28. | 30. 34, 37, 40. |
| 1, 8, 15. | 9, 72. | 13, 8, 3. |
| 9. 3. | 21. $8\frac{3}{4}$, $23\frac{1}{4}$. | 1, 3, 5. |
| 5. | $17\frac{1}{2}$, 70. | 31. 905. |
| 10. 8. | 22. $\frac{1}{4}$. | 3620. |
| 1. | $\frac{1}{4}$. | 32. 34. |
| 11. 21. | | 17. |
| 12. | | 33. 14. |

Exercise 97. Page 235.

8. $(a^2 + b^2)(c^2 + d^2)$. 9. $m^3 + 1$. 10. $4a^3(a - b)$.
 11. $\sqrt{\frac{a^2}{b}} + \sqrt{\frac{b^2}{a}}$. 12. $\frac{a+b}{2}$. 13. $\frac{a}{b^2} + \frac{b}{a^2}$.

Exercise 98. Page 239.

1. $x^{\frac{1}{2}}$; $x^{\frac{1}{2}}$; $x^{\frac{1}{2}}$; $a^{\frac{1}{2}}$; $a^{\frac{1}{2}}$; $a^{\frac{1}{2}}$; $a^{\frac{1}{2}}b^{\frac{1}{2}}$.
 2. $x^{\frac{1}{2}}y^{\frac{1}{2}}z$; $x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}}$; $a^{\frac{1}{2}}b^{\frac{1}{2}}c$; $5ab^{\frac{1}{2}}c^{\frac{1}{2}}x^2$.
 3. $\sqrt[3]{a^2}$; $\sqrt[6]{a^3b}$; $4\sqrt[6]{xy^5}$; $3\sqrt[3]{xy^2}$.
 4. $\frac{1}{a^2}$; $\frac{3}{xy^3}$; $\frac{6y}{x^3}$; $\frac{x^4}{y^5}$; $\frac{6xy^3}{ab^2}$.
 5. $3xyz^{-2}$; $x^{-3}y^{-4}z$; $ab^{-1}c^{-1}$; $a^{-3}b^2c^2$; $x^{-\frac{1}{2}}y^{\frac{1}{2}}$; $x^{-2}y^{-\frac{1}{2}}$.
 6. $a^{\frac{1}{2}}$; $b^{\frac{1}{2}}$; $c^{\frac{1}{2}}$; $d^{\frac{1}{2}}$. 12. $a^{\frac{1}{2}}$; $c^{\frac{1}{2}}$; $n^{-\frac{1}{2}}$; $a^{\frac{1}{2}}$.
 7. $m^{\frac{1}{2}}$; $n^{\frac{1}{2}}$; $a^{\frac{1}{2}}$; $\frac{1}{a^{\frac{1}{2}}}$. 13. a^{-1} ; c^{-1} ; m^{-2} ; n^{-1} ; x .
 8. a ; 1 ; $y^{\frac{1}{2}}$; $x^{\frac{1}{2}}$. 14. $p^{\frac{1}{2}}$; $q^{-\frac{1}{2}}$; $x^{\frac{1}{2}}y^{-1}$; $a^{-\frac{1}{2}}$.
 9. $a^{\frac{1}{2}}b^{\frac{1}{2}}c^{\frac{1}{2}}$; acd .
 10. $\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}z^{\frac{1}{2}}}$; $\frac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}}$. 15. $\frac{a}{8}$; $\frac{b^2}{9}$; $\frac{1}{32c^4}$; $\frac{4}{c^4}$.
 11. $\frac{1}{a^{\frac{1}{2}}}$; $\frac{y^{\frac{1}{2}}}{b^{\frac{1}{2}}x^{\frac{1}{2}}}$. 16. $\frac{27a^3b^{\frac{1}{2}}}{8}$; $\frac{64}{27a^6b^{\frac{1}{2}}}$; $\frac{a^2}{3^{\frac{1}{2}}}$; $\frac{125}{64}$.

Exercise 99. Page 240.

1. $x^{4p} + x^{2p}y^{2p} + y^{4p}$. 2. $x^{mn} - x^ny^n + x^{mn-n}y^{mn-n} - y^{mn}$.
 3. $x - 3x^{\frac{1}{2}} + 3x^{\frac{1}{2}} - 1$.
 4. $16a + 8a^{\frac{1}{2}}b^{\frac{1}{2}} + 10a^{\frac{1}{2}}b^{\frac{1}{2}} + 18a^{\frac{1}{2}}b^{\frac{1}{2}} - 8a^{\frac{1}{2}}b^{\frac{1}{2}}$
 $- 4a^{\frac{1}{2}}b^{\frac{1}{2}} - 5a^{\frac{1}{2}}b^{\frac{1}{2}} - 9b$.
 5. $1 + a^2b^{-2} + a^4b^{-4}$. 6. $a^4b^{-4} - 4 - 4a^{-2}b^2 - a^{-4}b^4$.
 7. $4x^5 - x^4 + 3x^3 + 2x^2 + x^{-1} + 1$.
 8. $x^{3n} + x^{2n}y^n + x^ny^{2n} + y^{3n}$.
 9. $x^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}} - x^{\frac{1}{2}}z^{\frac{1}{2}} + y^{\frac{1}{2}} - y^{\frac{1}{2}}z^{\frac{1}{2}} + z^{\frac{1}{2}}$. 10. $x^{\frac{1}{2}} + y^{\frac{1}{2}}$.
 11. $xy^{-1} + x^{-1}y$. 12. $a^{-2} + a^{-1}b^{-1} + b^{-2}$.

13. $16a^2b^{-2}$; $a - 2ab^{\frac{1}{2}} + b$; $a^2 + 2 + a^{-2}$;
 14. 4 ; 10 ; 4 ; $\frac{1}{4}$; $\frac{1}{3^2}$. $4ab^{\frac{1}{2}} - 4b + a^{-1}b^{\frac{1}{2}}$.
 15. $a^{\frac{1}{2}} - 3ab^{\frac{1}{2}} + 3a^{\frac{1}{2}}b^{\frac{1}{2}} - b$; $16x^{-4} + 32x^{-2} + 24 + 8x^2 + x^4$;
 $a^6b^{-6} - 6a^5b^{-4}y^{-1} - 15a^4b^{-2}y^{-2} - 20a^3y^{-3}$
 $+ 15a^2b^2y^{-4} - 6ab^4y^{-5} + b^6y^{-6}$.
 16. $3x^{-2} - 3x^{-1}y^{\frac{1}{2}} + y$. 17. $2x + 1 - 3x^{-1}$.
 18. 24 . 20. x^6 . 22. a^{m-1} . 24. 1 .
 19. x^6 . 21. $2ab^{\frac{1}{2}} + 11b$. 23. x^{pq} . 25. $x^{(q-1)p} - y^{(p-1)q}$.

Exercise 100. Page 243.

- $\sqrt{45}$; $\sqrt{189}$; $\sqrt{800}$; $\sqrt{a^4b^3c}$; $\sqrt[3]{x^5y^2}$.
- $\sqrt[4]{81x^3y^9}$; $\sqrt[5]{32x^6y}$; $\sqrt[4]{a^{15}b^2}$; $\sqrt[3]{27abc^7}$; $\sqrt{25a^3b^3c}$.
- $\sqrt{\frac{1}{4}}$; $\sqrt{244\frac{1}{4}}$; \sqrt{xy} .
- $xy^2\sqrt{z}$; $2a\sqrt{2ab}$; $3ay\sqrt[3]{2ax^2}$; $2\sqrt{6}$; $5a^2d\sqrt{5d}$.
- $10\sqrt[3]{a}$; $2xy^2\sqrt[3]{20xy}$; $3m^3n^3\sqrt[3]{4n}$; $7a^5b^5\sqrt[3]{4b}$.
- $(a-b)\sqrt[3]{a}$; $5(a-b)\sqrt{2}$.
- $4ac\sqrt[4]{5ab^2c^2}$; $42\sqrt{11x}$; $27y\sqrt[3]{3x^2z}$; $55\sqrt{6}$.
- $\frac{1}{8}\sqrt{5}$; $\frac{3}{4}\sqrt{3}$; $\frac{5}{4}\sqrt{2}$; $\frac{3}{4}\sqrt{58}$; $\sqrt[3]{4}$.
- $\frac{1}{z}\sqrt[3]{2xy^2z^2}$; $\frac{1}{8}\sqrt[3]{20}$; $\frac{1}{2b}\sqrt[4]{8ab}$; $\frac{a}{2cy^2}\sqrt{bcxy}$.
- $2\frac{2}{3}\sqrt{5}$; $1\frac{2}{3}\sqrt{21}$; $\frac{x^{\frac{1}{2}}z^{\frac{1}{2}}}{y^{\frac{1}{2}}}$; $\frac{a^{\frac{1}{2}}b^3}{c}$.
- $abx^{\frac{1}{2}}$; $2a^2b^{\frac{1}{2}}x$; $15a^{\frac{1}{2}}b^3y^{\frac{1}{2}}$.
- 7.071065; 42.42639; 0.707107; 0.1414213.

Exercise 101. Page 245.

- $3\sqrt{7}$. 4. $24\sqrt{3}$; $\sqrt{6}$. 7. $1\frac{1}{3}\sqrt{30}$.
- $6\sqrt{7}$; $5\sqrt{10}$; $9\sqrt{3}$. 5. $5\frac{2}{3}\sqrt{7}$; 2. 8. $4\frac{2}{3}$.
- $5\sqrt[3]{3}$; $4\sqrt[3]{4}$; $3\sqrt[3]{5}$. 6. $\frac{2}{3}\sqrt{3}$; $1\frac{1}{3}\sqrt{15}$. 9. $2\frac{2}{3}\sqrt[3]{4}$.

Exercise 102. Page 246.

1. $3\sqrt{2}$; $\frac{4}{3}\sqrt[4]{4}$; $2\sqrt[3]{3}$.
2. $\sqrt[3]{1\frac{1}{8}}$; $\sqrt{\frac{3}{8}}$.
3. $3\sqrt[3]{7}$; $4\sqrt{2}$; $2\sqrt[3]{22}$.
4. $3\sqrt{19}$; $5\sqrt[3]{2}$; $3\sqrt[3]{3}$.
5. $2ax\sqrt[6]{72ab^5}$; $a\sqrt[20]{a^3x^9y^{19}}$.
6. $\frac{3}{a^2}\sqrt[6]{2a^5b}$; $\frac{1}{b}\sqrt[12]{8a^{11}}$.
7. $\frac{1}{8}\sqrt[6]{225,000a^4b^4}$; 4.
8. $\frac{a}{xy}\sqrt[6]{b^5x^5y^2}$.
9. $6 - 30\sqrt{3} + 24\sqrt{10}$.
10. $\frac{1}{8}b^r$; $(2ab)^r$.
11. $a^3b^7\sqrt[7]{b^2}$; $a^{-1}b^{\frac{5}{2}}c^{\frac{1}{2}}d^{-1}$.

Exercise 103. Page 248.

1. $29\sqrt{3}$; $30\sqrt{10} + 164\sqrt{2}$.
2. $13\sqrt[3]{2}$; $33\sqrt[3]{2}$.
3. $48\sqrt{2}$; 0.
4. $\frac{8}{3}\sqrt{3}$; $\frac{4}{3}\sqrt{15}$.
5. $\left(\frac{a^2}{b^2} - \frac{ac}{bd} - \frac{ad}{bm}\right)\sqrt{bc}$; $\frac{2}{3}\sqrt{10}$.
6. $(a + 10b)\sqrt{ab}$.
7. $abc\sqrt[5]{ab^2c^3}$.
8. 0.
9. $8a^2\sqrt{3b}$; $9\sqrt[3]{3}$.
10. $\frac{1}{8}\sqrt{3a}$; $\sqrt[4]{27}$.
11. $\sqrt[3]{3}$; $\sqrt[4]{8}$; $\frac{1}{2}\sqrt[3]{2}$; $\sqrt[3]{2}$; $\sqrt[4]{3}$.
12. $\sqrt[5]{2}$; $\sqrt[5]{6}$; $\sqrt{2}$; $\sqrt{3}$; $\sqrt[3]{5}$; $\sqrt{7}$.
13. $\sqrt[3]{2x^2}$; $\sqrt[3]{3ab^2}$; $a\sqrt{2a}$; $2a^2$.
14. $2\sqrt[3]{2}$; $3\sqrt[3]{3}$; $4\sqrt[4]{2}$; $2\sqrt[3]{2}$.
15. a^{-4} ; x^{-3} ; $p^{\frac{1}{2}}$; $a^{\frac{1}{2}}$.
16. $a^3\sqrt{a} + 5a^2\sqrt{b} + 10ab\sqrt{a} + 10ab\sqrt{b} + 5b^2\sqrt{a} + b^3\sqrt{b}$;
 $m^2 + 3mx\sqrt[6]{m^2x^3} + 3x^3\sqrt[3]{m^2} + x^4\sqrt{x}$;
 $a^2\sqrt{a} - 10a^2\sqrt{b} + 40ab\sqrt{a} - 80ab\sqrt{b} + 80b^2\sqrt{a} - 32b^2\sqrt{b}$.

17. $64a^{12} - 96a^{10}\sqrt{a} + 60a^8 - 20a^7\sqrt{a} + 3\frac{2}{3}a^6 - \frac{8}{3}a^4\sqrt{a} + \frac{a^3}{64};$
 $16x^3\sqrt[5]{x} - 16x^2y^2\sqrt[5]{x^2} + 6xy^4\sqrt[5]{x^3} - y^6\sqrt[5]{x^4} + \frac{y^8}{16};$
 $64x^{12}y^{-6} - 192x^{10}y^{-4}\sqrt[3]{y^{-1}} + 240x^8y^{-2}\sqrt[3]{y^{-2}}$
 $- 160x^6y^{-1} + 60x^4\sqrt[3]{y^2} - 12x^2y^2\sqrt[3]{y} + y^4.$
18. $a^{\frac{1}{2}}b^{\frac{1}{2}} - \frac{5}{2}a^2b^{\frac{1}{2}}c + \frac{5}{2}a^{\frac{1}{2}}b^{\frac{1}{2}}c^2 - \frac{5}{4}ab^{-1}c^3 + \frac{5}{16}a^{\frac{1}{2}}b^{-1}c^4 - \frac{1}{32}b^{-\frac{1}{2}}c^5;$
 $\frac{a^{10}c^{-5}}{32} - \frac{5}{48}a^8c^{-\frac{1}{2}} + \frac{5}{36}a^6c^{-2} - \frac{5}{24}a^4c^{-\frac{3}{2}} + \frac{5}{16}a^2c^{-\frac{5}{2}} - \frac{c^{\frac{1}{2}}}{243};$
 $a^8b^4 - 2a^5b^{\frac{1}{2}} + \frac{3}{2}a^2b^3 - \frac{a^{-1}b^{\frac{1}{2}}}{2} + \frac{a^{-4}b^2}{16}.$
19. $a^3b^{-3}c^{\frac{1}{2}}d^{-\frac{1}{2}} - 3a^2b^{-2} + 3ab^{-1}c^{-\frac{1}{2}}d^{\frac{1}{2}} - c^{-3}d^3;$
 $a^{2n} - 4a^n + 6 - 4a^{-n} + a^{-2n};$
 $16a^4b^{-8} - \frac{3}{2}a^{\frac{1}{2}}b^{-5} + \frac{8}{3}a^3b^{-2} - \frac{8}{27}a^{\frac{1}{2}}b + \frac{a^2b^4}{81}.$
20. $\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}c^{\frac{1}{2}}} - \frac{1}{b^2c^{\frac{1}{2}}} + \frac{c^{\frac{1}{2}}}{3a^{\frac{1}{2}}b^{\frac{1}{2}}} - \frac{c^{\frac{1}{2}}}{27a^{\frac{1}{2}}b^3};$
 $\frac{a^{\frac{1}{2}}}{8b^2} - \frac{9a}{4b^{\frac{1}{2}}} + \frac{27}{2}a^{\frac{1}{2}}b^{\frac{1}{2}} - 27b^{\frac{1}{2}};$
 $\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}} - \frac{3a^2}{2b^{\frac{1}{2}}} + \frac{3}{4a^{\frac{1}{2}}b^{\frac{1}{2}}} - \frac{b^{\frac{1}{2}}}{8a^3}.$
21. $x^{2m} + 3x^my^n + y^{2n}.$
22. $1 + 2x^{-1} - 3x^{-2} + 4x^{-1}.$

Exercise 104. Page 250.

1. $\frac{3}{2}(\sqrt{7} - \sqrt{5}); \frac{1}{2}(2\sqrt{5} + \sqrt{6}); 5\sqrt{2} - 6; 30 + 12\sqrt{6}.$
2. $\frac{a(\sqrt{b} + \sqrt{c})}{b - c}; \frac{(a+b)(a+\sqrt{b})}{a^2 - b}; \frac{2(x-y)\sqrt{xy} + 3xy}{y(x-4y)}.$
3. 1.154.....; 1.216.....; 3.576.....; 19.726.....

Exercise 105. Page 251.

1. 19; 11.
2. $6\sqrt{-3}$; $-ax\sqrt{by}$.
3. $b-a$; $-a^4b^4\sqrt{b}$.
4. $-2\sqrt{5}$; $-2(3+13\sqrt{-15})$.
5. $\frac{x}{y}$; $-\sqrt{-1}$; $-\sqrt{-a}$.
6. 2; $-\sqrt{-3}$; $\frac{1}{2}$.

Exercise 106. Page 254.

1. $3+\sqrt{5}$.
2. $2\sqrt{3}+\sqrt{5}$.
3. $\sqrt{7}+\sqrt{3}$.
4. $\sqrt{11}+\sqrt{5}$.
5. $\sqrt{7}-\sqrt{2}$.
6. $2(\sqrt{3}-\sqrt{2})$.
7. $\sqrt{6}-\sqrt{3}$.
8. $7-3\sqrt{5}$.
9. $\sqrt{10}-\sqrt{3}$.
10. $3-\sqrt{2}$.
11. $2\sqrt{3}-\sqrt{2}$.
12. $2\sqrt{5}-3\sqrt{2}$.
13. $3\sqrt{11}-2$.
14. $3\sqrt{5}-2\sqrt{3}$.
15. $\frac{1}{2}\sqrt{10}-1$.
16. $\sqrt{a+b}+\sqrt{a-b}$.
17. $\sqrt{a^2-b^2}-b$.
18. $3\sqrt{7}-2\sqrt{6}$.
19. $(a-b)-2\sqrt{ab}$.

Exercise 107. Page 256.

1. 9.
2. 4, 0.
3. $2\pm\frac{1}{3}\sqrt{6}$.
4. 6, $2\frac{3}{4}$.
5. 16, 1.
6. 221, 5.
7. 5, $1\frac{2}{3}$.
8. $\frac{3}{4}$.
9. ± 24 .
10. ± 5 , $\pm 3\sqrt{2}$.
11. $\frac{1}{2}$, $-1\frac{1}{2}$.
12. -28 , 0.
13. 12, 3.
14. 1, $\frac{1}{2}$.
15. 1.
16. $\pm\sqrt{65}$, $\pm\sqrt{5}$.
17. $1\frac{1}{4}$.
18. $\pm 2a$.
19. 2, 0.
20. $2\frac{8}{21}$, $\frac{2}{3}$.
21. $2a$.
22. $-2a$.
23. $\pm 9\sqrt{2}$.
24. 4.
25. ± 8 .
26. $2\sqrt[3]{18}$, 4.
27. $2\frac{1}{2}$, $\frac{2}{3}$.
28. $4a^2$, a^2 .
29. $\pm 3\frac{2}{3}$, $\pm 1\frac{7}{8}\sqrt{-13}$.

Exercise 108. Page 259.

1. $7, \pm 4, -1$.
2. $\frac{2}{3}, -1\frac{1}{2}, \frac{1}{12}(-13 \pm \sqrt{313})$.
3. $2\frac{1}{2}, -1, \frac{1}{4}(3 \pm \sqrt{-15})$.
4. $\frac{a+2b}{2a}, \frac{2b-9a}{2a}$.
5. $1, -\frac{1}{2}, \frac{1}{4}(1 \pm \frac{1}{8}\sqrt{41})$.
6. $4, 1, 3\frac{2}{3}, 1\frac{1}{3}$.
7. $1, \frac{1}{2}(-3 \pm \sqrt{5})$.
8. $\pm 4, \pm \sqrt{23}$.
9. $\frac{1}{2}(-1 \pm \sqrt{5}), \frac{1}{2}(-1 \pm \frac{1}{3}\sqrt{58})$.
10. $6\frac{29}{100}$.
11. $4, 1$.
12. $1\frac{1}{2}\frac{1}{3}$.
13. $\frac{1}{2}(1 \pm \sqrt{-3}, \pm \sqrt{-1})$.
14. $5, -2\frac{1}{2}, \frac{1}{4}(5 \pm \sqrt{97})$.
15. $\frac{1}{4}, -\frac{3}{8}, \frac{1}{8}(1 \pm \sqrt{33})$.
16. $\frac{4a^2+b^4}{8b^3}$.
17. $\frac{1}{8}, -2\frac{1}{8}, 0$.
18. $\pm 2\sqrt{2}, \pm \frac{1}{4}\sqrt{-2}, [\frac{1}{4}(5 \pm \sqrt{41})]^{\frac{1}{2}}$.
19. $-8\frac{1}{2}, 5, \frac{1}{8}(4 \mp 3\sqrt{109})$.
20. $\frac{1}{2}(a+b+1) \mp \frac{1}{4}\sqrt{4a+1} \mp \frac{1}{4}\sqrt{4b+1}$.
21. $\pm 2, \pm 1, \frac{1}{2}(\pm \sqrt{6} \pm \sqrt{2}), \pm 1, \pm 2, \frac{1}{2}(\pm \sqrt{6} \mp \sqrt{2})$.
22. $6, 2, \frac{1}{2}(-9 \pm \sqrt{33})$.
23. $\frac{1}{4b}(a^2+b^2) \pm \frac{1}{4b}\sqrt{10a^2b^2-3a^4-3b^4}$.
24. $3, 2, \frac{1}{2}(5 \pm \sqrt{61}), -1 \pm \sqrt{10}, -1 \pm \sqrt{-5}$.
25. $3 \pm 2\sqrt{2}, 0, (3 \pm 2\sqrt{2})^2, 0$.
26. $6, -4\frac{1}{2}, 12, -9$.
27. $\pm 16, \pm 4, \pm 4, \pm 16$.
28. $\frac{9a}{2}(a \pm \sqrt{a^2+a}), \frac{a}{2}(a \pm \sqrt{a^2+9a})$.
29. $\pm 2, \pm \frac{1}{2}$.
30. $9, 0$.

Exercise 109. Page 264.

- | | | |
|-------------|------------------|------------------|
| 1. 0.7781. | 12. 2.6232. | 23. 0.0970. |
| 2. 1.1761. | 13. 1.0791. | 24. 1.5741. |
| 3. 1.3222. | 14. 1.7781. | 25. 0.3222. |
| 4. 1.1461. | 15. 1.8751. | 26. 1.2040. |
| 5. 1.5441. | 16. 0.8751. | 27. 8.7481 — 10. |
| 6. 0.9542. | 17. 8.3222 — 10. | 28. 9.7993 — 10. |
| 7. 0.9030. | 18. 9.5441 — 10. | 29. 0.2431. |
| 8. 1.6902. | 19. 7.5441 — 10. | 30. 2.0212. |
| 9. 1.3980. | 20. 7.6020 — 10. | 31. 8.0212 — 10. |
| 10. 1.4771. | 21. 8.6990 — 10. | 32. 0.0212. |
| 11. 1.6232. | 22. 1.0970. | |

Exercise 110. Page 266.

- | | | | |
|------------|-------------|-------------|-------------|
| 1. 0.9030. | 7. 0.1003. | 13. 0.2258. | 19. 2.9579. |
| 2. 1.3980. | 8. 0.3495. | 14. 0.4660. | 20. 0.6361. |
| 3. 3.3804. | 9. 0.0596. | 15. 0.2045. | 21. 2.4465. |
| 4. 3.8168. | 10. 0.1690. | 16. 0.2415. | 22. 0.4730. |
| 5. 2.5353. | 11. 0.1398. | 17. 1.1650. | 23. 0.5243. |
| 6. 3.4950. | 12. 0.0768. | 18. 0.3904. | 24. 1.3280. |
| | | | 25. 1.1569. |

Exercise 111. Page 268.

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|-----------------|------------------|------------------|
| 1. 9.6020 — 10. | 11. 0.1761. | 21. 0.4559. |
| 2. 9.4559 — 10. | 12. 1.1461. | 22. 8.8539 — 10. |
| 3. 9.7781 — 10. | 13. 8.2219 — 10. | 23. 7.6320 — 10. |
| 4. 9.6320 — 10. | 14. 7.3980 — 10. | 24. 7.5441 — 10. |
| 5. 9.8539 — 10. | 15. 8.1461 — 10. | 25. 5.1707 — 10. |
| 6. 0.1461. | 16. 1.8539. | 26. 4.8293. |
| 7. 0.2219. | 17. 2.6320. | 27. 5.9333. |
| 8. 0.3980. | 18. 6.6320 — 10. | 28. 4.1040. |
| 9. 0.3680. | 19. 1.2219. | 29. 2.8627 — 10. |
| 10. 0.5441. | 20. 9.5441 — 10. | 30. 7.1373. |

Exercise 112. Page 273.

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|------------------|------------------------|----------------|
| 1. 1.7782. | 30. 0.695. | 59. 12940. |
| 2. 2.0043. | 31. 0.001051. | 60. 99.82. |
| 3. 2.9996. | 32. 0.0455. | 61. 2.381. |
| 4. 3.9957. | 33. 41.64. | 62. 2.155. |
| 5. 3.7329. | 34. 0.02989. | 63. 282.6. |
| 6. 3.5775. | 35. 0.3131. | 64. 1.706. |
| 7. 4.7350. | 36. 8.294. | 65. 146.8. |
| 8. 4.9581. | 37. — 28.47. | 66. 1.938. |
| 9. 4.0000. | 38. — 0.3144. | 67. 1.913. |
| 10. 4.0004. | 39. 0.0104. | 68. 1.616. |
| 11. 4.8490. | 40. — 660,600. | 69. 9.218. |
| 12. 4.0802. | 41. 1.308. | 70. 2.468. |
| 13. 7.9943 — 10. | 42. 0.6307. | 71. 5.487. |
| 14. 9.9430 — 10. | 43. 105.4. | 72. 1.523. |
| 15. 0.0000. | 44. 0.7167. | 73. 0.2356. |
| 16. 2.9430. | 45. — 111.3. | 74. 1.293. |
| 17. 1.8686. | 46. 0.001086. | 75. 0.7397. |
| 18. 0.8494. | 47. 0.7277. | 76. 0.02558. |
| 19. 8.9528 — 10. | 48. 0.2084. | 77. 1.568. |
| 20. 1.9990. | 49. 6.963. | 78. 6.955. |
| 21. 17,730. | 50. — 5.219. | 79. 0.002431. |
| 22. 4295. | 51. 221.5. | 80. -0.3525. |
| 23. 339.6. | 52. 1.416. | 81. 0.0005218. |
| 24. 88.8. | 53. 2.257. | 82. 0.01804. |
| 25. 53,910. | 54. 15.78. | 83. 33.04. |
| 26. 93.98. | 55. 0.2387. | 84. 28.39. |
| 27. 0.7586. | 56. 0.03593. | 85. 12.58. |
| 28. 1.9. | 57. 7.212. | 86. 0.5096. |
| 29. 0.1099. | 58. 0.0001174. | |
| | 87. 0.000000000004936. | |

Exercise 113. Page 282.

1. 24:35. 4. 15:16; 23:25; 10:11; 3:4. 6. 4:15.
 2. 10:9. 5. $a+b:a-b > a^2+b^2:a^2-b^2$. 7. 2:3.
 8. $a^4+a^2x^2+x^4:a^4-x^4$. 10. 1:1. 13. 6, 8.
 9. $x^2-11x+28:x^2$. 11. 18, 27. 14. 4:1.
 15. 1:5. 17. $\frac{np-mq}{q-p}$.
 16. 26, 40, A to B;
 52, 60, B to C. 18. 1240, 680 rods.

Exercise 114. Page 290.

16. 3, $\frac{1}{2}$. 18. $\frac{b(a+b)^2}{(a-b)^2}$. 19. ± 9 . ± 3 .
 17. $a+b, \frac{1}{2}(a-b)$. 20. 1. 4.
 21. ± 2 . 25. 440 and 352 yards per min.
 22. 8:7. 26. 45 and 30 miles per hour.
 23. A's, \$200; B's, \$150. 27. 70.
 24. 6. 28. 3:7; $3\frac{3}{20}:7\frac{7}{25}$.

Exercise 115. Page 296.

1. $9\frac{3}{4}$. 6. $64X^2=9Y^3$. 10. Vel., $161\frac{1}{2}$; dis., $402\frac{1}{2}$.
 2. $\frac{3}{4}$. 7. $X^2=\frac{108}{Y^3}$. 11. $9\frac{1}{4}$. 15. 1 inch.
 3. 15. 12. $20\frac{1}{4}$ feet. 16. 1:1.
 4. 5. 13. $516\frac{1}{2}$ oz. 17. 10.
 5. $3\frac{3}{4}$. 14. 87.1.... dys. 18. 18 feet.

Exercise 116. Page 305.

1. 53. 13. -29. $a+21b$. -1. 0. 2. $4\frac{1}{8}$.
 3. $7\frac{1}{2}$; 6; a^2 . 4. $5\frac{1}{2}$, 10, $14\frac{1}{2}$; $\frac{1}{8}$, $4\frac{3}{8}$, $8\frac{3}{8}$, $12\frac{3}{8}$.

5. 25th. 7. 185. 35. $\frac{an}{2}(3n-1)$. — 28. $336\frac{3}{4}$.
 6. — 7.
 8. $1, 2\frac{1}{2}, 3\frac{1}{2}, 5\frac{1}{2}, 6\frac{3}{4}$, 8. 9. 9. 10. — 10. 11. 6, 14.
 12. 6. 15. 4, 15, 26, 18. 5, 7, 9.
 13. 12. 16. 3, 5, 7; or 7, 5, 3. 19. $-\frac{3}{4}, \frac{1}{4}, 1\frac{1}{4}$.
 14. 5. 17. 8. 20. 14.
 21. 10 days. 22. 234. 23. 2, 6, 10, 14; or 14, 10, 6, 2.
 25. 7 days. 27. 3, 6, 15; or 15, 12, 3.
 26. $6433\frac{1}{4}$ feet. 28. 10 seconds; $\frac{1}{4}$ the distance.

Exercise 117. Page 311.

1. 1458. 96. $1\frac{3}{8}$. — 128. x^{14} . $20\frac{1}{4}a^5m^4$.
 2. $6x^2y^2\sqrt{15z}$. 3. 4. 4. 20, 50; 28, 56, 112. 5. 5th.
 6. 3; 192. 7. 765. 547. $15\frac{3}{4}$. 1953.1. $\frac{205m}{256}$.
 8. 10%. 9. 3. 10. $1\frac{1}{2}\frac{3}{8}$. 12. $\frac{1}{2}, 1, 2, 4$. 13. $\frac{3}{4}\frac{2}{3}$.
 14. 12, 18, 27. 16. 0.59049. 18. 1, 3, 9.
 15. $1, 2\frac{1}{4}$. 17. 2, 4, 6, 9. 19. 49, 1.
 20. 1, 3, 9, 27. 21. 139.
 22. 8. $1\frac{1}{2}$. $\frac{1}{8}$. $\frac{5}{7}$. $\frac{3}{10}$. $1\frac{1}{2}$. $\frac{1}{9}$. $\frac{8}{9}$. $\frac{11}{10}$. $\frac{11}{8}$.

Exercise 118. Page 315.

1. $2\frac{3}{4}, 3, 4, 6$. 2. 9, 1; or $\frac{1}{3}$, — $7\frac{1}{2}$. 3. 12.
 4. $13\frac{1}{3}, 14\frac{2}{7}, \dots, 17\frac{1}{7}, 18\frac{6}{13}$. 5. 6th.
 6. $6, 6\frac{3}{4}, 6\frac{5}{7}, 7\frac{5}{13}$. 7. 8, 2. 8. $\frac{a+b}{2}, \sqrt{ab}, \frac{2ab}{a+b}$.
 9. 104, 234. 10. 2, 3, 6.

For answers to Exercises 119 and 120 of Elements of Algebra, see answers to 120 and 121 of this book; and for answers to Exercise 121 of Elements of Algebra, see Appendix.

Exercise 119. Page 338.

- | | | |
|---------------------|-------------|----------------|
| 1. 454,053,600. | 8. 1956. | 15. 33. |
| 2. 6. | 9. 109,600. | *16. 720. |
| 3. 6. | 10. 2520. | 17. 400,400. |
| 4. 12. | 11. 420. | 18. 6160. |
| 5. 4845; 969; 3876. | 12. 5040. | 19. 161,700. |
| 6. 95,040. | 13. 244. | 20. 1,368,000. |
| 7. 5040; 144. | 14. 6720. | 21. 15. |
-
- | | | |
|---|-----------------------------------|-----------|
| 22. $\frac{\overline{m} \times \overline{n}}{(\overline{r} \times \overline{m-r})(\overline{s} \times \overline{n-s})}$. | 23. $\frac{\overline{9}}{2}$. | 25. 5775. |
| | 24. 11; 3. | 26. 36. |
| 27. $\frac{\overline{90}}{\overline{20} \overline{70}}; \frac{\overline{89}}{\overline{19} \overline{70}}$. | 28. 80. | 30. 325. |
| | 29. $\overline{9} \overline{2}$. | 31. 645. |
| | | 32. 7. |
| | | 33. 52. |

Exercise 120. Page 345.

- $1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$.
- $x^8 - 24x^7 + 252x^6 - 1512x^5 + 5670x^4 - 13,608x^3$
 $+ 20,412x^2 - 17,496x + 6561$.
- $16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$.
- $8 - 12x + 6x^2 - x^3$.
- $1 - \frac{15y}{4} + \frac{45y^2}{8} - \frac{135y^3}{32} + \frac{405y^4}{256} - \frac{243y^5}{1024}$.
- $1 - 3x + 4x^2 - \frac{28x^3}{9} + \frac{14x^4}{9} - \frac{14x^5}{27} + \frac{28x^6}{243}$
- $-14,080,000x^9y^3. \quad -\frac{4x^7}{243} + \frac{x^8}{729} - \frac{x^9}{19683}$.

* If one man occupies a particular place, the number of ways is $\overline{5}$ or 120. If no one occupies a particular place, the number of ways is $\overline{6}$ or 720.

8. $\frac{35x^4y^8}{1944}$.
 9. $-1365a^{13}x^{11}$.
 10. $-115,200x^{11}y^{16}$.
 11. 70.
 12. -252 .
 13. $-\frac{35x}{y}; +\frac{35y}{x}$.
 14. $\frac{n(n-1)\dots(n-r+2)}{1 \times 2 \times \dots(r-1)} (2a)^{n-r+1}x^{r-1}$.
 15. $\frac{n(n-1)\dots(n-r+2)}{1 \times 2 \times \dots(r-1)} (2a)^{r-1}x^{n-r+1}$.
 16. $\frac{n(n-1)\dots(n-r-2)}{1 \times 2 \times \dots(r+3)} a^{n-r-3}x^{r+3}$.
 17. $\frac{|2n}{(|n|^2)} a^n x^n$.
 18. 0.

Exercise 121. Page 350.

1. $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \dots$
 2. $1 + \frac{2}{3}x - \frac{1}{3}x^2 + \frac{4}{81}x^3 - \dots$
 3. $a^{\frac{1}{2}} \left\{ 1 + \frac{3x}{4a} - \frac{3x^2}{32a^2} + \frac{5x^3}{128a^3} - \dots \right\}$.
 4. $1 + 4x + 10x^2 + 20x^3 + \dots$

If n be an *integer*, in the expansions of $(1+x)^n$ or $(1-x)^{-n}$, the coefficients are all positive; of $(1-x)^n$ or $(1+x)^{-n}$, they are alternately positive and negative.

5. $a^5 - \frac{5a^3x^2}{2} + \frac{15ax^4}{8} - \frac{5x^6}{16a} + \dots$
 6. $x^{-3} - \frac{3}{2}x^{-4}y + \frac{1}{8}x^{-5}y^2 - \frac{3}{16}x^{-6}y^3 + \dots$
 7. $(2x)^{-1} \left\{ 1 + \frac{3}{8}x^{-1}y + \frac{4}{128}x^{-2}y^2 + \frac{5}{1024}x^{-3}y^3 + \dots \right\}$.
 8. $1 - x - 2x^2 - 6x^3 - \dots$

9. $\frac{1}{8a^3} \left\{ 1 + \frac{9x}{8a} + \frac{135x^2}{128a^2} + \frac{945x^3}{1024a^3} + \dots \right\}.$
10. $1 + \frac{5}{2}y + \frac{5^2}{8}y^2 + \frac{5^3}{4}y^3 + \dots$
11. $1 + \frac{2}{3}x + \frac{8}{9}x^2 - \frac{1}{3}x^3 + \dots$
12. $1 - \frac{2}{3}x + \frac{4}{9}x^2 - \frac{8}{27}x^3 + \dots$
13. $(-1)^r \frac{1 \times 3 \times 5 \dots (2r-5)}{r-1 \times 2^{r-1}} a^{\frac{3-2r}{2}} x^{r-1}.$

The factor $(-1)^r$ is introduced because the r th term will be positive or negative according as r is even or odd.

14. $\frac{r(r+1)}{1 \times 2} a^{-r-2} x^{r-1}.$
15. 8.06224. 16. 1.01099. 17. 2.002224.
18. $1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \dots$ 19. 351.

Exercise 122. Page 358.

1. $\frac{1}{8}, \frac{1}{3}, \frac{2}{3}.$ 2. $\frac{3}{4}.$ 3. $\frac{1}{17}.$ 4. $\frac{5}{7}, \frac{2}{7}.$
5. 7 to 2 against the event. 6. $\frac{1}{2}.$
7. $\frac{1}{2}.$ 10. $\frac{2}{3}.$ 14. Their chances are $\frac{1}{4}, \frac{1}{4}, \frac{1}{2}.$
8. $\frac{1}{4} \frac{1}{16}.$ 12. $\frac{2}{8}.$ 15. \$40. 17. 34 to 43 in favor of C.
9. $\frac{1}{5}.$ 13. $\frac{2}{3}.$ 16. 2 to 3. 18. $\frac{7}{12}, \frac{5}{12}; \frac{7}{22}, \frac{5}{22}, \frac{3}{8}.$
19. $\frac{1}{55} \frac{1}{25}.$ 20. $\frac{1}{17}, \frac{1}{16}.$ 23. $\frac{1}{12}.$ 24. $\frac{1}{15}, \frac{1}{10}, \frac{3}{12}.$

Exercise 123. Page 371.

1. $\frac{3}{4}.$ 5. 5 to 3 against. 9. $\frac{33}{1000}.$
2. $\frac{11}{16}.$ 7 to 1 against. 10. $\frac{1}{6}.$
3. $\frac{n}{2^n}.$ 6. 6 to 1. 11. $\frac{8}{125}, \frac{12}{125}, \frac{36}{125}.$
4. $1 - (\frac{3}{8})^3.$ 7. $\frac{2}{15}.$ 12. $\frac{1}{30}, \frac{1}{10}, \frac{1}{6}.$
8. 49 to 15 against.

13. $\frac{96}{825}, \frac{113}{825}, \frac{898}{825}, \frac{258}{825}$.
 14. 7 to 2.
 15. $\frac{5}{8}$.
 16. $\frac{35}{84}$.
 17. $\frac{1}{10}$.
 18. $\frac{|n|}{n^n}$.
 19. $\frac{12383}{10000}$.
 20. $1 - (\frac{35}{36})^3$.
 21. $\frac{15}{108}$.
 22. $\frac{125}{3888}, \frac{23}{648}, \frac{3875}{3888}$.
 23. $\frac{14}{72}, \frac{1}{25}, \frac{1}{27}, \frac{7}{45}$.
 24. $\frac{80}{243}$.
 25. $1 - \frac{1053}{5^5}$.
 26. 131 to 112.
 27. $\frac{3497 \times 5^5}{8^8}$.
 28. $\frac{125}{216}$.
 29. As 16 : 12 : 9.
 30. $\$0.37\frac{1}{2}$. 7 to 1.
 31. He may expect a loss of \$0.25.
 32. $\$6.83\frac{11}{168}, \$13.05\frac{125}{168}, \$3.86\frac{4}{21}$.
 33. $+\$2\frac{2}{3}, -\$2\frac{2}{3}, -\$2\frac{1}{3}; +\$2\frac{1}{2}, -\$2, -\$2\frac{3}{4}$.

Exercise 124. Page 382.

1. Between 48 and 49.
 2. (i.) $\frac{B-A}{Ar}$;
 (ii.) $\frac{\log B - \log A}{\log R}$.
 3. \$0.00563.
 4. $4.310 \times \$A$.
 5. \$444, nearly.
 6. \$20,270, nearly.
 7. 5.35%.
 8. \$34.40.
 9. 22.
 10. 115, nearly.
 11. 5%, nearly.
 12. 4.2%, nearly.
 13. \$32.30.
 14. 4%.
 15. \$7298.
 16. A little more than 9 years.
 17. A little less than 17 years.
 18. \$969, nearly.
 19. 51.37.
 20. 4.7.
 21. $2\frac{1}{8}\%$.
 22. 2,428,000.
 23. \$27,563.

Exercise 125. Page 392.

$$1. \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}}}}}}$$

$$2 + \frac{1}{4}; \quad 3 + \frac{1}{3} + \frac{1}{6}; \quad 9 + \frac{1}{1} + \frac{1}{3} + \frac{1}{1} + \frac{1}{18}.$$

The fifth convergents are $\frac{1}{14}$; $\frac{14}{13}$; $\frac{2888}{1197}$; $\frac{3870}{1197}$; $\frac{970}{99}$.

$$2. \quad \frac{1}{5 + \frac{1}{2 + \frac{1}{7 + \frac{1}{3}}}}$$

$$\frac{1}{2 + \frac{1}{4 + \frac{1}{6 + \frac{1}{8 + \frac{1}{10}}}}}$$

$$2 + \frac{1}{4 + \frac{1}{6 + \frac{1}{8}}}$$

$$5 + \frac{1}{3 + \frac{1}{1 + \frac{1}{3 + \frac{1}{5 + \frac{1}{7}}}}}$$

The third convergents are $\frac{15}{82}$; $\frac{457}{204}$; $\frac{25}{13}$.

3. $4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{8} + \text{etc.}}}}}}}}$

$$5 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{10} + \text{etc.}}}}$$

$$7 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{14} + \text{etc.}}}}$$

4. $\frac{117}{37}$; $\frac{244}{83}$; $\frac{268}{65}$; $\frac{140}{33}$; $\frac{161}{38}$.

7. $\frac{79}{98}$.

5. $\frac{79}{98}$.

8. $\frac{98}{79}$.

6. $\frac{98}{79}$.

9. $\frac{27}{4}$; $\frac{333}{108}$; $\frac{355}{113}$.

10. 15 and 16, 61 and 65, 76 and 81, etc.

11. $\sqrt{15}$; $\frac{1}{4}(\sqrt{197}-13)$; $\frac{1}{4}(\sqrt{1093}-13)$.

12. Limit of error = 0.00012.

13. $\frac{1}{4}$, $\frac{7}{28}$, $\frac{8}{33}$, $\frac{39}{168}$,

14. $\frac{1}{4}$, $\frac{3}{4}$, $\frac{13}{17}$, $\frac{250}{397}$,

Exercise 126. Page 411.

1. $\frac{1}{2} + \frac{3x}{4} + \frac{9x^2}{8} + \frac{27x^3}{16} + \dots$

2. $\frac{1}{2} - \frac{x}{4} + \frac{3x^2}{8} - \frac{9x^3}{16} + \dots$

3. $\frac{3}{4} + \frac{x}{16} + \frac{3x^2}{64} + \frac{9x^3}{256} + \dots$

4. $1 - x^2 - x^3 + x^5 + \dots$ 7. $4x + 2x^2 - 8x^3 - 22x^4 - \dots$

5. $1 + 2x + x^2 - 4x^3 - \dots$ 8. $y - y^2 + y^3 - y^4 + \dots$

6. $5 - 17x + 56x^2 - 185x^3 + \dots$ 9. $y + 2y^2 + 5y^3 + 14y^4 + \dots$

10. $y + \frac{y^3}{3} - \frac{y^5}{5} + \dots$

11. $y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \dots$

12. $\frac{1+2x}{1-x+x^2}$; $\frac{3-7x}{1-3x+x^2}$; $\frac{3}{4+x-5x^2}$.

13. $\frac{3}{x+4} + \frac{4}{x-5}$

15. $\frac{8}{3(x-5)} - \frac{1}{3(2x-1)}$

14. $\frac{6}{x+3} - \frac{6}{x+4}$

16. $\frac{4}{7(x+2)} + \frac{3}{7(x-5)}$

17. $\frac{1}{x-1} - \frac{x+2}{x^2+x+1}$

18. $\frac{3}{4x} - \frac{1}{8(x-2)} + \frac{3}{8(x+2)}$

19. $\frac{4}{25x} - \frac{4}{5x^2} + \frac{71}{25(x+5)}$

20. $\frac{11}{3(x-1)} + \frac{2}{(x-1)^2} + \frac{10}{3(x+2)}$

21. $\frac{10}{3(x+1)} - \frac{4x+7}{3(x^2-x+1)}$

Exercise 127. Page 420.

1. 77323.

3. 631.

5. 2.425.

7. 58.

2. 11860.

4. 120.

6. 766.5.

8. 255.

9. $3n(n+1)(n+3)$.

10. $\frac{1}{8}n(n+1)(4n-1)$.

Exercise 128. Page 423.

1. 11.

4. 460.

7. 4095.

10. 5525.

2. 11879.

5. 1540; 2870.

8. 1840.

11. 4970.

3. 5610.

6. 190.

9. 25.

Exercise 129. Page 426.

1. $\frac{1}{3} \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3} \right); \frac{11}{18}$

2. $\frac{1}{8} \left(1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} \right)$

+ $\frac{1}{8} \left(-\frac{1}{n+1} - \frac{1}{n+2} + \frac{1}{n+3} + \frac{1}{n+4} \right)$.

$$3. \frac{1}{16} \left(1 - \frac{2}{2} + \frac{1}{2} \right) + \frac{1}{16} \left(-\frac{1}{n+1} + \frac{1}{n+2} \right); \frac{1}{32}$$

$$4. -\frac{1}{2} + \frac{4}{3} + \frac{1}{n+2} - \frac{4}{n+3}; \frac{5}{6}$$

$$5. \frac{1}{2} \left(1 - \frac{2}{2} + \frac{1}{2} \right) + \frac{1}{2} \left(-\frac{1}{n+1} - \frac{1}{n+2} \right).$$

Exercise 130. Page 431.

- | | | |
|--------------------------------------|-------------|----------|
| 1. $2r^4 + 3r^3 + 8r^2 + 7r + 6.$ | 3. 12. | 5. 1111. |
| 2. $2r^4 + 7r^3 + 20r^2 + 23r + 20.$ | 4. 9. | 6. 3. |
| 7. 20.2444. | 8. 2709316. | |

Exercise 131. Page 453.

- | | |
|-----------------------------|--|
| 1. $x=7, y=-2.$ | 12. $\begin{cases} x=10, y=2. \\ x=2, y=10. \end{cases}$ |
| 2. $x=4, y=2.$ | 13. $\begin{cases} x=13, y=3. \\ x=-3, y=-13. \end{cases}$ |
| 3. $x=1, y=-1.$ | 14. $\begin{cases} x=8, y=\frac{3}{2}. \\ x=-3, y=-4. \end{cases}$ |
| 4. $x=\frac{1}{2}, y=5.$ | 15. $\begin{cases} x=9, y=4. \\ x=4, y=9. \end{cases}$ |
| 5. $x=8, y=4.$ | 16. $\begin{cases} x=-12, y=14. \\ x=-3, y=-4. \end{cases}$ |
| 6. $x=2, y=1.$ | 17. $x=2, y=\frac{3}{5}.$ |
| 7. $x=1, y=-1.$ | |
| 8. $x=29, y=23.$ | |
| 9. $x=2, y=-1.$ | |
| 10. $x=-1, y=-\frac{1}{2}.$ | |
| 11. $x=\frac{1}{2}, y=1.$ | |
| 18. $x=6, y=9.$ | |

Exercise 134. Page 462.

1. $x^3 - x^2 - 44x + 84 = 0.$
2. $x^4 - x^3 - 13x^2 + x + 12 = 0.$
3. $x^5 + 6x^4 - 13x^3 - 78x^2 + 36x + 216 = 0.$
4. $x^3 + 0.075x^2 - 0.105x + 0.01 = 0.$
5. $x^3 - 11x^2 + 40x - 50 = 0.$

Exercise 135. Page 463.

1. $x^3 - 3x^2 - 10x + 24 = 0$.
2. $x^3 + 6x^2 - 9x - 14 = 0$.
3. $x^3 - 4x = 0$.
4. $x^3 - 18x^2 + 108x - 216 = 0$.
5. $x^4 - 5x^2 + 4 = 0$.
6. $4x^4 - 17x^2 + 4 = 0$.

Exercise 136. Page 464.

1. $-3, -8$.
2. $-6, -17$.
3. $\pm a, \pm a\sqrt{3}$.
4. $-2, 1 \pm \sqrt{-3}, \pm 2\sqrt{-1}$.
5. $\frac{2}{3}, -\frac{1}{4}$.
6. $\frac{1}{2}(-3 \pm \sqrt{17}), 1, \frac{1}{2}$.
7. $\frac{1}{2}b, \frac{1}{2}b$.
8. $\pm 2, 1 \pm \sqrt{-3}, -1 \pm \sqrt{-3}$.
9. $\frac{1}{2}, \pm \sqrt{-1}$.
10. $\frac{1}{2}, \frac{1}{2}(-1 \pm \sqrt{-3}), 0$.

Exercise 137. Page 465.

1. $4, \frac{1}{2}(-7 \pm \sqrt{37})$.
2. $-2, 3 \pm \sqrt{5}$.
3. $3, 2 \pm \sqrt{-3}$.
4. $3, 3, 2$.
5. $-5, \frac{1}{2}(5 \pm \sqrt{29})$.
6. $-4, \frac{1}{2}(7 \pm \sqrt{153})$.
7. $2, -3, -8$.
8. $2, \pm 5$.
9. $1, 9, -7$.
10. $-3, 7, -4$.

Exercise 138. Page 467.

1. $1, 2, 3, -4$.
2. $2, -1, 2 \pm 2\sqrt{2}$.
3. $2, 4, -1 \pm \sqrt{-3}$.
4. $3 \pm \sqrt{2}, 3 \pm \sqrt{2}$.
5. $1, 1, 2, -4$.
6. $4, -2, -1 \pm \sqrt{-1}$.
7. $2, 3, 1 \pm \sqrt{-6}$.
8. $2, -2, -1 \pm \sqrt{6}$.
9. $13, -1, -1, -7$.
10. $\frac{1}{2}(2 \pm \sqrt{-11}), \frac{1}{2}(7 \pm \sqrt{37})$.

Exercise 140. Page 475.

1. Last derivative = 2.
2. Last derivative = 6.
3. Last derivative = 24.
4. Last derivative = 120.
5. Last derivative = 24.

Exercise 141. Page 477.

1. 1, 1, 6.
2. 2, 2, 3.
3. -1, -1, -1, 3.
4. 3, 3, -4.
5. 3, 3, 3, -2.
6. 2, 2, 2, -6.
7. 5, 5, 1, -1.
8. 5, 5, 5, -2, -2.
9. 1, 1, 1, $-\frac{1}{2}(1 \pm \sqrt{-11})$.
10. 1, 1, -4, -4.

Exercise 142. Page 479.

1. $x^3 + 2x^2 - 18x + 18 = 0$.
2. $x^3 + 10x^2 - 42x - 576 = 0$.
3. $x^4 - 6x^3 - 1350x^2 - 18,000x + 162,000 = 0$.
4. $x^5 + 3x^4 + 24x^3 - 72x^2 + 1296x - 23,328 = 0$.
5. $x^4 - 8x^2 + 4x - 224 = 0$.

Exercise 143. Page 482.

1. $y^3 - 8y^2 + 12y + 9 = 0$.
2. $y^4 + 2y^3 - 8y^2 - 6y + 15 = 0$.
3. $y^3 + 4y^2 - 15y - 18 = 0$.
4. $y^3 - 5y = 0$.
5. $y^4 + 17y^3 + 92y^2 + 172y + 96 = 0$.
6. $y^4 + 4.8y^3 - 17.86y^2 - 56.688y + 90.4761 = 0$.
7. $y^4 + 1.2y^3 + 1.54y^2 - 2.292y + 3.1981 = 0$.
8. $y^5 + 3.5y^4 + 4.50y^3 + 5.75y^2 + 1.8125y - 16.15625 = 0$.
9. $y^5 - 8y^4 + 20y^3 - 19y^2 - 2y + 20 = 0$.
10. $y^6 + 0.2y^5 + 1.60y^4 - 1.640y^3 + 2.6240y^2 - 3.70208y + 5.138944 = 0$.

Exercise 145. Page 487.

1. 637.
2. -596.
3. 0.
4. -499.
5. 5225.
6. 82.
7. 347.
8. 60146.
9. -182.
10. 11659.

Exercise 146. Page 491.

1. $1.+, 0.4+, -1.+.$ 6. $1.+, 5.+, -0.6+.$
2. $2.+, -0.6+, -1.+.$ 7. $-1.+, -3.+, -4.+.$
3. $-1.+, 1.+, 4.+.$ 8. $1.+, 6.+, 7.+.$
4. $1.+, 1.+, -3.+.$ 9. $0.7+, 2.+, -0.7+, 5.+.$
5. $-7.+, 3.+, 1.+.$ 10. $2.+, 0.4+, 0.6+, -3.+.$

Exercise 147. Page 499.

1. $2.833066+.$ 6. $8.414454+.$ 11. $43.$
2. $4.264429+.$ 7. $4.898989+.$ 12. $742.826359+.$
3. $1.157450+.$ 8. $32.865378+.$ 13. $7.103575+.$
4. $2.645751+.$ 9. $624.$ 14. $-4.532088+.$
5. $-3.907378+.$ 10. $29.$ 15. $-0.732050+.$

EXAMPLE IN § 622 : $3.353, 3.415.$ **Exercise 148. Page 504**

1. $\pm 1, \frac{1}{2}(-7 \pm 3\sqrt{5}).$
2. $\frac{1}{4}(\sqrt{5}-1 \pm \sqrt{-10-2\sqrt{5}}),$
 $-\frac{1}{4}(\sqrt{5}+1 \mp \sqrt{-10+2\sqrt{5}}).$
3. $\pm 1, \frac{1}{4}(3 + \sqrt{-7} \pm \sqrt{-14+6\sqrt{-7}}),$
 $\frac{1}{4}(3 - \sqrt{-7} \pm \sqrt{-14-6\sqrt{-7}}).$
4. $2 \pm \sqrt{3}, \frac{1}{2}(1 \pm \sqrt{-3}).$ 6. $-1, 2 \pm \sqrt{3}, \frac{1}{2}(1 \pm \sqrt{-3}).$
5. $1, 1, \frac{1}{4}(1 \pm \sqrt{-15}).$ 7. $3 \pm 2\sqrt{2}, 2 \pm \sqrt{3}.$
8. $-1, \frac{1}{2}(1-m \pm \sqrt{m^2-2m-3}).$
9. $-1, \frac{1}{4}(1 + \sqrt{5} \pm \sqrt{2\sqrt{5}-10}),$
 $\frac{1}{4}(1 - \sqrt{5} \pm \sqrt{-2\sqrt{5}-10}).$
10. $1, \pm \sqrt{-1}, \frac{1}{8}(-1 \pm \sqrt{-35}).$

Exercise 149. Page 505.

1. $2.438+.$ 5. $0.951+.$ 9. $1.774.$ 13. 20 years.
2. $2.096+.$ 6. $1.900+.$ 10. $2.925.$ 14. 8 years.
3. $2.872+.$ 7. $0.569+.$ 11. 15 years. 15. 17 years.
4. $2.970+.$ 8. $0.801+.$ 12. 9 years.

Exercise 150. Page 507.

1. $-2, -5, -5.$
2. $10, \frac{1}{2}(11 \pm 5\sqrt{-3}).$
3. $2, 2 \pm \sqrt{-1}.$
4. $1, -2 \pm 3\sqrt{-1}.$
5. $-6, 3 \pm 5\sqrt{-3}.$
6. $8, -4 \pm 3\sqrt{-3}.$
7. $1, 1, -2.$
8. $-11, \frac{1}{2}(11 \pm \sqrt{-123}).$

Exercise 151. Page 510.

1. $1.15418, -0.57709 \pm 1.9998\sqrt{-1}.$
2. $-0.41813, 0.20906 \pm 2.6704\sqrt{-1}.$
3. $-3.2264, 1.6132 \pm 0.89838\sqrt{-1}.$
4. $-2.4567, -1.2283 \pm 0.72557\sqrt{-1}.$
5. $1, 1.5616, -2.5616.$

**APPENDIX.****Exercise 121. (Elements of Algebra.)**

1. $\frac{p^{25}}{32} + \frac{15p^{20}y^4}{16} + \frac{45p^{15}y^8}{4} + \frac{135p^{10}y^{12}}{2} + \frac{405p^5y^{16}}{2} + 243y^{20}.$
2. $\frac{32a^{15}}{243} + \frac{20a^{12}b^2}{27} + \frac{5a^9b^4}{3} + \frac{15a^6b^6}{8} + \frac{135a^3b^8}{128} + \frac{243b^{10}}{1024}.$
3. $a^{10}b^{15} + 10a^{11}b^{12}x^4 + 40a^{12}b^{11}x^8 + 80a^{13}b^9x^{12} + 80a^{14}b^7x^{16} + 32a^{15}b^5x^{20}.$
4. $\frac{128a^7b^{14}}{2187} - \frac{112a^8b^{12}y}{243} + \frac{14a^9b^{10}y^2}{9} - \frac{35a^{10}b^8y^3}{12} + \frac{105a^{11}b^6y^4}{32}$
 $- \frac{567a^{12}b^4y^5}{256} + \frac{1701a^{13}b^2y^6}{2048} - \frac{2187a^{14}y^7}{16384}.$
5. $a^5\sqrt{a} + 7a^3x + 21a^2\sqrt{a} \times x^2 + 35a^2x^3 + 35a\sqrt{a} \times x^4 + 21ax^5$
 $+ 7\sqrt{a} \times x^6 + x^7.$
6. $16b^4 - 64b^3\sqrt{2b} \times m + 224b^2m^2 - 224b^2\sqrt{2b} \times m^3 + 280b^2m^4$
 $- 112b\sqrt{2b} \times m^5 + 56bm^6 - 8\sqrt{2b} \times m^7 + m^8.$

7. $27c^3\sqrt{3c} + 378c^3a + 756c^2\sqrt{3c} \times a^2 + 2520c^2a^3 + 1680c\sqrt{3c} \times a^4 + 2016ca^5 + 448\sqrt{3c} \times a^6 + 128a^7.$
8. $\frac{a^4}{16} - 3a^3\sqrt{\frac{1}{3}a} \times y + \frac{63a^3y^2}{2} - 378a^2\sqrt{\frac{1}{3}a} \times y^3 + \frac{2835a^2y^4}{2} - 6804a\sqrt{\frac{1}{3}a} \times y^5 + 10,206ay^6 - 17,496\sqrt{\frac{1}{3}a} \times y^7 + 6561y^8.$
9. $64e^3 + 192ae^2\sqrt{e} + 240a^2e^2 + 160a^3e\sqrt{e} + 60a^4e + 12a^5\sqrt{e} + a^6.$
10. $\frac{64a^6}{729} + \frac{64a^5\sqrt{2x}}{81} + \frac{160a^4x}{27} + \frac{320a^3x\sqrt{2x}}{27} + \frac{80a^2x^2}{3} + 16ax^2\sqrt{2x} + 8x^3.$
11. $\frac{243a^5}{1024} - \frac{405a^4\sqrt{\frac{1}{3}x}}{256} + \frac{135a^3x}{64} - \frac{45a^2x\sqrt{\frac{1}{3}x}}{16} + \frac{15ax^2}{16} - \frac{x^2\sqrt{\frac{1}{3}x}}{4}.$
12. $64a^6 - 576a^5\sqrt{y} + 2160a^4y - 4320a^3y\sqrt{y} + 4860a^2y^2 - 2916ay^2\sqrt{y} + 729y^3.$
13. $a^{14} + \frac{7a^{12}\sqrt{z}}{2} + \frac{21a^{10}z}{4} + \frac{35a^8z\sqrt{z}}{8} + \frac{35a^6z^2}{16} + \frac{21a^4z^2\sqrt{z}}{32} + \frac{7a^2z^3}{64} + \frac{z^3\sqrt{z}}{128}.$
14. $b^4 - 8b^3\sqrt{by} + 28b^2y - 56b^2y\sqrt{by} + 70b^2y^2 - 56by^2\sqrt{by} + 28by^3 - 8y^3\sqrt{by} + y^4.$
15. $8c^3 + 24c^2\sqrt{6cx} + 180c^2x + 120cx\sqrt{6cx} + 270cx^2 + 54x^2\sqrt{6cx} + 27x^3.$
16. 8,200,192 a^6 . 18. 326,592 x^{10} . 20. $\frac{1547a^{11}x^9}{256}.$
17. 352,716 $a^{11}d^{10}$. 19. - 12,033,222,880 y^{81} . 21. - 165 a^4x^2 .
22. $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots$
 $1 - \frac{1}{9} + \frac{1}{9^2} - \frac{1}{9^3} + \frac{1}{9^4} - \frac{1}{9^5} + \dots = 0.9.$
23. $(1+x)^{\frac{1}{3}} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \frac{7x^5}{256} - \dots = 1 + \frac{3}{16} - \frac{9}{512}$
 $+ \frac{27}{8192} - \frac{405}{524288} + \frac{1701}{8388608} - \dots = 1.1726.$
24. $(1+x)^{-\frac{1}{3}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128} - \frac{63x^5}{256} + \dots = 0.998503367 \dots$

$$25. a^{-5} - 5a^{-6}x + 15a^{-7}x^2 - 35a^{-8}x^3 + \dots$$

$$26. a^{-6} + 6a^{-7}x + 21a^{-8}x^2 + 56a^{-9}x^3 + \dots$$

$$27. \frac{1}{64b^6} + \frac{3y}{64b^7} + \frac{21y^2}{256b^8} + \frac{7y^3}{64b^9} + \dots$$

$$28. \frac{512}{c^9} - \frac{9216z}{c^{10}} + \frac{92160z^2}{c^{11}} - \frac{675840z^3}{c^{12}} + \dots$$

$$29. \frac{1}{a^9} - \frac{9x}{2a^{10}} + \frac{45x^2}{4a^{11}} - \frac{165x^3}{8a^{12}} + \dots$$

$$30. \frac{1}{a^{10}} - \frac{5x^3}{a^{12}} + \frac{15x^6}{a^{14}} - \frac{35x^9}{a^{16}} + \dots$$

$$31. \frac{1}{a^3} + \frac{6x^2}{a^3\sqrt{a}} + \frac{21x^4}{a^4} + \frac{56x^6}{a^4\sqrt{a}} + \dots$$

$$32. \sqrt{b} + \frac{h}{2\sqrt{b}} - \frac{h^2}{8b\sqrt{b}} + \frac{h^3}{16b^2\sqrt{b}} - \dots$$

$$33. \sqrt[4]{b} - \frac{x}{4\sqrt[4]{b^3}} - \frac{3x^2}{32b\sqrt[4]{b^3}} - \frac{7x^3}{128b^2\sqrt[4]{b^3}} - \dots$$

$$34. x + \frac{a}{2x} - \frac{a^2}{8x^3} + \frac{a^3}{16x^5} - \dots$$

$$35. a - \frac{1}{2a} - \frac{1}{8a^3} - \frac{1}{16a^5} - \dots$$

$$36. 1 + \frac{a}{9} - \frac{4a^2}{81} + \frac{68a^3}{2187} - \dots$$

$$37. \frac{1}{a\sqrt{a}} - \frac{1}{2a^4\sqrt{a}} + \frac{3}{8a^7\sqrt{a}} - \frac{5}{16a^{10}\sqrt{a}} + \dots$$

$$38. \frac{1}{x} - \frac{a}{2x^3} + \frac{3a^2}{8x^5} - \frac{5a^3}{16x^7} + \dots$$

$$39. 1 + \frac{x^5}{3} + \frac{2x^{10}}{9} + \frac{14x^{15}}{81} + \dots \quad 41. \frac{1}{8} - \frac{3h}{256} + \frac{3h^2}{2048} - \frac{13h^3}{65536} + \dots$$

$$40. 1 - \frac{2d}{5} + \frac{12d^2}{25} - \frac{88d^3}{125} + \dots \quad 42. \frac{1}{27} + \frac{x^2}{81} + \frac{5x^4}{1458} + \frac{35x^6}{39366} + \dots$$



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